

- 1 If a sequence is convergent, then it is bounded from above.
- If a sequence is convergent, then it is eventually monotonic.
- **③** If a sequence diverges and is increasing, then there exists *n* ∈  $\mathbb{N}$  such that  $a_n > 100$ .
- (1) If  $\lim_{n \to \infty} a_n = L$ , then  $a_n < L + 1$  for all n.
- If a sequence is non-decreasing and non-increasing, then it is convergent.
- If a sequence isn't decreasing and isn't increasing, then it is convergent.

Let *f* be a function with domain at least  $[0, \infty)$ . We define a sequence  $(a_n)_{n \in \mathbb{N}}$  as  $a_n = f(n)$ . Let  $L \in \mathbb{R}$ .

- **1** IF  $\lim_{x \to \infty} f(x) = L$ , THEN  $\lim_{n \to \infty} a_n = L$ .
- **2** IF  $\lim_{n \to \infty} a_n = L$ , THEN  $\lim_{x \to \infty} f(x) = L$ .
- **3** IF  $\lim_{n \to \infty} a_n = L$ , THEN  $\lim_{n \to \infty} a_{n+1} = L$ .

Let *f* be a function with domain  $[0, \infty)$ . We define the sequence  $(a_n)_{n>0}$  by  $a_n = f(n)$ .

**1** IF *f* is increasing, THEN  $(a_n)_n$  is increasing.

**2** IF  $(a_n)_n$  is increasing, THEN *f* is increasing.

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Beware: do not confuse sequences defined by

 $R_n = f(n)$ 

(as in the two previous slides) and sequences defined by induction

$$R_{n+1} = f(R_n)$$

(as last Monday).

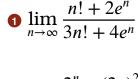
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## Big Theorem – True or False

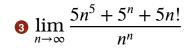
Let  $(a_n)_{n \in \mathbb{N}}$  and  $(b_n)_{n \in \mathbb{N}}$  be positive sequences.

- **1** IF  $a_n \ll b_n$ , THEN  $\forall m \in \mathbb{N}, a_m < b_m$ .
- **2** IF  $a_n \ll b_n$ , THEN  $\exists m \in \mathbb{N}$  s.t.  $a_m < b_m$ .
- $\textbf{3} \text{ IF } a_n << b_n, \text{ THEN } \exists n_0 \in \mathbb{N} \text{ s.t. } \forall m \in \mathbb{N}, \ m \ge n_0 \implies a_m < b_m.$
- **4** IF  $\forall m \in \mathbb{N}$ ,  $a_m < b_m$ , THEN  $a_n \ll b_n$ .
- **(3)** IF  $\exists m \in \mathbb{N}$  s.t.  $a_m < b_m$ , THEN  $a_n << b_n$ .
- **6** IF  $\exists n_0 \in \mathbb{N}$  s.t.  $\forall m \in \mathbb{N}, m \ge n_0 \implies a_m < b_m$ , THEN  $a_n \ll b_n$ .





$$2 \lim_{n \to \infty} \frac{2^n + (2n)^2}{2^{n+1} + n^2}$$



- Write a version of the Squeeze Theorem for convergent sequences.
- Write a comparison theorem for sequences divergent to +∞.
- Homework: prove them!

A sequence  $(a_n)_{n \in \mathbb{N}}$  is called *2-increasing* when

 $\forall n \in \mathbb{N}, \quad a_n < a_{n+2}.$ 

Construct a sequence that is 2-increasing but not increasing.

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Convergent sequence of integers – Homework

Prove that if a sequence of integers is convergent then it is eventually constant.

## Composition law – Homework

Write a proof for the following Theorem

## Theorem

Let  $(a_n)_{n \in \mathbb{N}}$  be a sequence. Let  $L \in \mathbb{R}$ . • IF  $\begin{cases} \lim_{n \to +\infty} a_n = L \\ f \text{ is continuous at } L \end{cases}$ • THEN  $\lim_{n \to +\infty} f(a_n) = f(L)$ . Prove, directly from the definition of limit, that

$$\lim_{n\to\infty}\frac{n^2}{n^2+1}=1.$$

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