Volumes – 2

UNIVERSITY OF TORONTO

February 8th, 2019

For next week

For Monday (Feb 11), watch the videos:

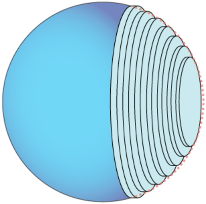
- Sequences and first properties: 11.1, 11.2, 11.3, 11.4

For Wednesday (Feb 13), watch the videos:

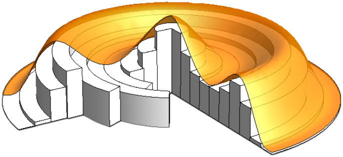
- Theorems about sequences: 11.5, 11.6, 11.7, 11.8

For today:

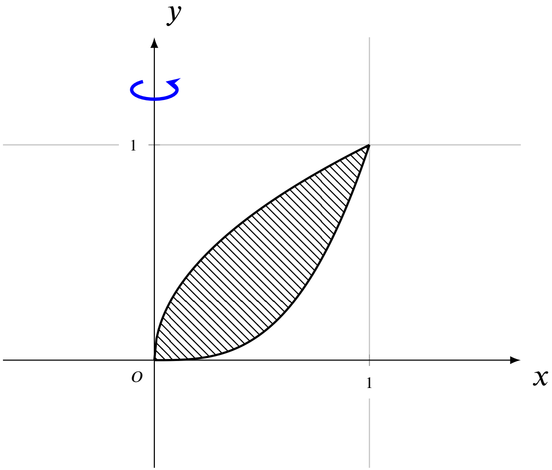
Slice method:



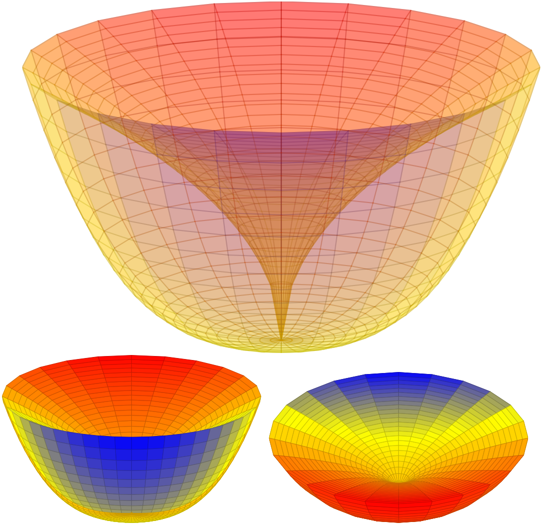
Shell method:



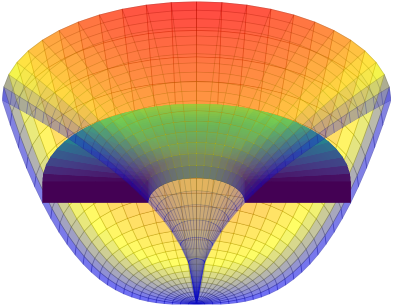
Let R be the region in the first quadrant bounded between the curves $y = x^3$ and $y = \sqrt{x}$.
We are interested in the solid of revolution obtained by revolving R around the y-axis.

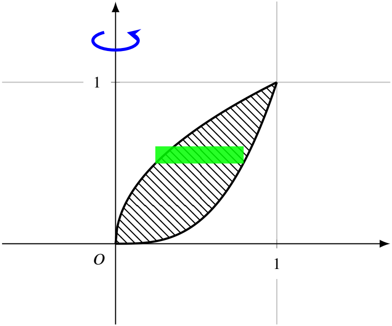


The solid:



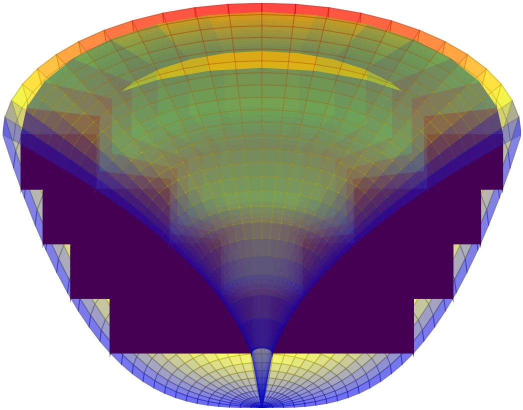
Slicing method:



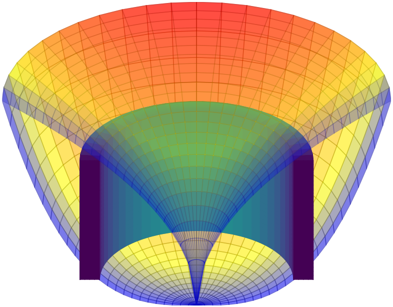


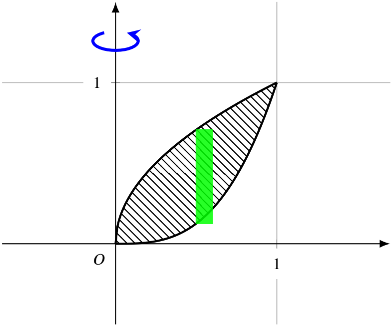
The section is perpendicular to the axis of revolution.
We integrate along the axis parallel to the axis of revolution.

Slicing method:



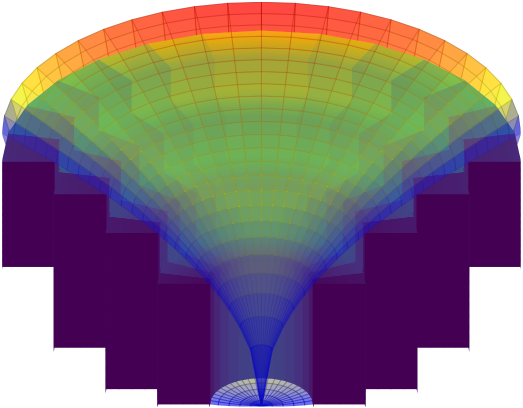
Shell method:





The section is parallel to the axis of revolution.
We integrate along the axis perpendicular to the axis of revolution.

Shell method:



A first computation with shell methods

Let R be the region in the first quadrant bounded between the curves $y = x^3$ and $y = \sqrt{x}$.

Using the shell method, compute the volume of the solid of revolution obtained by revolving R around the y-axis.

Is it compatible with the result obtained last Monday using the slicing method?

A first computation with shell methods

Let R be the region in the first quadrant bounded between the curves $y = x^3$ and $y = \sqrt{x}$.

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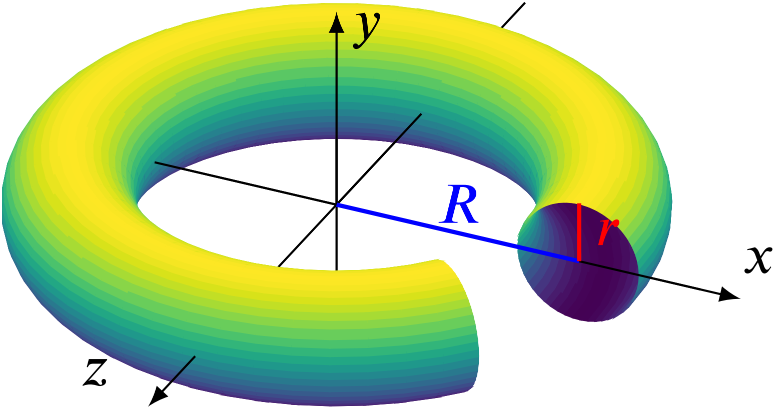
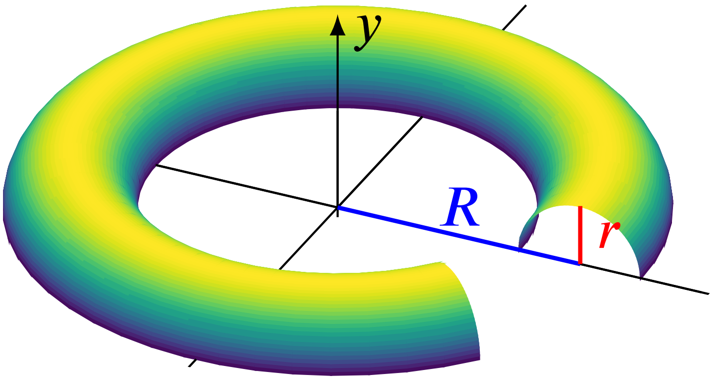
Is it compatible with the result obtained last Monday using the slicing method?

Torus

Let $r, R \in \mathbb{R}$ such that $R > r > 0$.
The solid obtained by revolving the disk of radius r centered at $(R, 0)$ around the y-axis is called a *torus*.

- Draw the circle of radius r centered at $(R, 0)$.
- Draw the torus obtained above.
- Recall the equation of the circle of radius r centered at (a, b).
- Find a formula for the volume of the torus using the shell method.


y
R
r
x
z


y
R
r

An equation for volumes using the shell method – Homework

Let $f : [a, b] \to \mathbb{R}$ be a continuous positive function where $0 \le a < b$.

Let R be the region in the first quadrant enclosed between the graph of f and the x-axis.

Find a formula for the volume of the solid of revolution obtained by rotation the region R around the y-axis.