# Trigonometric and rational functions

UNIVERSITY OF TORONTO

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# For next week

For Monday (Feb 4), watch the videos:

- Volumes: 10.1

For Wednesday (Feb 6), watch the videos:

- Volumes: 10.2
- Sequences: 11.1, 11.2

# Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

*Hint:* You will need

- $\frac{d}{dx}[\tan x] = \ldots$
- $\frac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

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# Practice: Integrals with trigonometric functions

Compute the following antiderivatives.
Once you get them to a form from where it is easy to finish, you may stop: we are only interested in the method!

1. $\displaystyle\int \sin^{10} x \cos x \, dx$
2. $\displaystyle\int \sin^{10} x \cos^3 x \, dx$
3. $\displaystyle\int e^{\cos x} \cos x \sin^5 x \, dx$
4. $\displaystyle\int \cos^2 x \, dx$
5. $\displaystyle\int \sin^4 x \, dx$
6. $\displaystyle\int \csc x \, dx$

## Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

# Rational integrals

1. Compute $\displaystyle\int \frac{1}{x+a}\,dx$
2. Reduce to common denominator $\displaystyle\frac{2}{x} - \frac{3}{x+3}$
3. Compute $\displaystyle\int \frac{-x+6}{x^2+3x}\,dx$
4. Compute $\displaystyle\int \frac{1}{x^2+3x}\,dx$
5. Compute $\displaystyle\int \frac{1}{x^3-x}\,dx$

# Repeated factors

1. Compute $\displaystyle\int \frac{1}{(x+1)^n}\,dx$ for $n > 1$
2. Compute $\displaystyle\int \frac{x}{(x+1)^2}\,dx$
3. Compute $\displaystyle\int \frac{3x+2}{(x+1)^2}\,dx$
4. Compute $\displaystyle\int \frac{x^3}{(x+1)^2}\,dx$

# Irreducible quadratics

1. Compute $\displaystyle\int \frac{1}{x^2+1}\,dx$ and $\displaystyle\int \frac{x}{x^2+1}\,dx$.
2. Compute $\displaystyle\int \frac{2x+3}{x^2+1}\,dx$
3. Compute $\displaystyle\int \frac{x^2}{x^2+1}\,dx$
4. Compute $\displaystyle\int \frac{1}{x^2+x+1}\,dx$

# A reduction formula – *Homework*

Let $I_n = \int_0^{2\pi} \sin^n x \, dx$.

1. Compute $I_0$ and $I_1$.

2. Starting with $I_n$, use integration by parts. Then use the main trig identity to obtain an equation involving $I_n$ and $I_{n-2}$.

3. Compute $I_8$ and $I_{55}$.