

INTEGRATION BY SUBSTITUTION

January 23rd, 2019

Computation practice: integration by substitution

Use substitutions to compute:

$$① \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$② \int e^x \cos(e^x) dx$$

$$③ \int \cot x dx$$

$$④ \int x e^{-x^2} dx$$

$$⑤ \int \frac{e^{2x}}{\sqrt{e^x + 1}} dx$$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

$$\text{Compute } I = \int_0^2 \sqrt{x^3 + 1} x^2 dx$$

Wrong answer

Substitution: $u = x^3 + 1$, $du = 3x^2 dx$.

$$\begin{aligned} I &= \frac{1}{3} \int_0^2 \sqrt{x^3 + 1} (3x^2 dx) &= \frac{1}{3} \int_0^2 u^{1/2} du \\ &= \frac{1}{3} \frac{2}{3} u^{3/2} \Big|_0^2 &= \frac{1}{9} (x^3 + 1)^{2/3} \Big|_0^2 \\ &= \frac{2}{9} (2^3 + 1)^{3/2} - \frac{2}{9} (0 + 1)^{3/2} &= \frac{52}{9} \end{aligned}$$

A different kind of substitution

Compute

$$\int_0^1 \sqrt{1 - x^2} dx$$

using the substitution $x = \sin \theta$.