MAT137Y1 – LEC0501

Calculus!

The Fundamental Theorem of Calculus Part 2

UNIVERSITY OF TORONTO

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For next week

For Monday (Jan 28), watch the videos:

- Integration by parts: (9.5), 9.6, (9.7), (9.8), (9.9)

For Wednesday (Jan 30), watch the videos:

- Integration of trig functions: 9.10, (9.11), (9.12)
- Integration of rational functions: 9.15, (9.16), (9.17)

What is wrong?

Let $f(x) = \frac{1}{x^4}$ and $F(x) = -\frac{1}{3x^3}$.
Notice that $F' = f$.
Hence, according to FTC-2, we have

$$\int_{-1}^{1} \frac{1}{x^4} dx = \left. \frac{-1}{3x^3} \right|_{-1}^{1} = -\frac{2}{3}$$

However, x^4 is always positive and $-1 < 1$, so the integral should be positive.

Definite integrals

Justify that the following integrals are well defined and compute them:

1. $\displaystyle\int_1^2 x^3 dx$
2. $\displaystyle\int_0^1 \left[e^x + e^{-x} - \cos(2x)\right] dx$
3. $\displaystyle\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx$
4. $\displaystyle\int_{\pi/4}^{\pi/3} \sec^2 x \, dx$
5. $\displaystyle\int_1^2 \left[\frac{d}{dx}\left(\frac{\sin^2 x}{1+\arctan^2 x + e^{-x^2}}\right)\right] dx$

Riemann sums

Compute the following limits

1. $\lim_{n\to+\infty} \frac{1}{n} \sum_{k=1}^{n} \tan \frac{k}{n}$
2. $\lim_{n\to+\infty} \sum_{k=1}^{n} \frac{n}{n^2+k^2}$
3. $\lim_{n\to+\infty} \prod_{k=1}^{n} \left(1+\frac{k}{n}\right)^{\frac{1}{n}}$

Hints:

- $\frac{d}{dx}\left(-\ln|\cos(x)|\right) = \tan(x)$
- $\frac{d}{dx}\left(x\ln(x) - x\right) = \ln(x)$

Area

Compute the area of the bounded region between $y = \frac{x^2}{2}$ and $y = \frac{1}{1+x^2}$.