The Fundamental Theorem of Calculus
Part 1

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For next lecture

For Wednesday (Jan 23), watch the videos:

- FTC – 2: 8.5, 8.6, 8.7
- Integration by substitution: 9.1, (9.2), (9.3), (9.4)

True or False?

Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function.
Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function.
Assume that $\forall x \in \mathbb{R},\ f'(x) = g(x)$.
Which of the following statements must be true?

1. $f(x) = \int_0^x g(t)dt$.
2. If $f(0) = 0$, then $f(x) = \int_0^x g(t)dt$.
3. If $g(0) = 0$, then $f(x) = \int_0^x g(t)dt$.
4. There exists $C \in \mathbb{R}$ such that $f(x) = C + \int_0^x g(t)dt$.
5. There exists $C \in \mathbb{R}$ such that $f(x) = C + \int_1^x g(t)dt$.

True or False?

We want to find a function H with domain $\mathbb{R}$ such that $H(1) = -2$ and such that $H'(x) = e^{\sin x}$ for all x.

Decide whether each of the following statements is true or false.

1. The function $H(x) = \int_0^x e^{\sin t} dt$ is a solution.
2. For any $C \in \mathbb{R}$, the function $H(x) = \int_0^x e^{\sin t} dt + C$ is a solution.
3. There is a $C \in \mathbb{R}$ such that $H(x) = \int_0^x e^{\sin t} dt + C$ is a solution.
4. The function $H(x) = \int_1^x e^{\sin t} dt - 2$ is a solution.
5. There is more than one solution.

A generalized version of FTC–1

Exercise

Let f be a continuous function with domain $\mathbb{R}$.
Let u, v be differentiable functions with domain $\mathbb{R}$.
Set

$$H(x) = \int_{u(x)}^{v(x)} f(t)dt$$

Justify that H is differentiable on $\mathbb{R}$ and find a formula for

$$H'(x)$$

in terms of f, u, v, f', u', v'.

Assume f is a continuous function with domain $\mathbb{R}$ that satisfies:

$$\forall x \in \mathbb{R},\ \int_0^x e^t f(t)dt = \frac{\sin x}{x^2+1}$$

Find an explicit expression for $f(x)$.