Part 2: Antiderivatives and indefinite integrals

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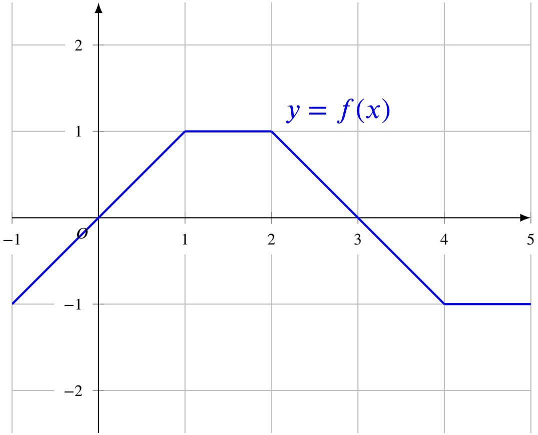
Initial Value Problem

Find a function $f : \mathbb{R} \to \mathbb{R}$ such that

- For every $x \in \mathbb{R}$, $f''(x) = \sin x + x^2$,
- $f'(0) = 5$,
- $f(0) = 7$.

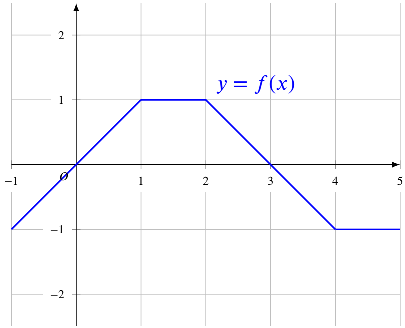
Towards FTC



Compute:

1. $\displaystyle\int_0^1 f(t)dt$
2. $\displaystyle\int_0^2 f(t)dt$
3. $\displaystyle\int_0^3 f(t)dt$
4. $\displaystyle\int_0^4 f(t)dt$
5. $\displaystyle\int_0^5 f(t)dt$

Towards FTC (continued)



Call $F(x) = \int_0^x f(t)dt$. This is a new function.

- Sketch the graph of $y = F(x)$.
- Using the graph you just sketched, sketch the graph of $y = F'(x)$.

Trig-exp antiderivatives

1. Compute

$$\frac{d}{dx}\left[e^x \sin x\right], \qquad \frac{d}{dx}\left[e^x \cos x\right].$$

2. Use the previous answer to Compute

$$\int e^x \sin x \, dx, \qquad \int e^x \cos x \, dx.$$

A harder antiderivative

1. Compute

$$\frac{d}{dx}[\arctan x], \qquad \frac{d}{dx}\left[\frac{x}{1+x^2}\right].$$

2. Use the previous answer to compute

$$\int \frac{1}{\left(1+x^2\right)^2}\,dx$$