Part 1:
Properties of the integral

UNIVERSITY OF TORONTO

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For next week

For Monday (Jan 21), watch the videos:

- FTC – 1: 8.3, 8.4

For Wednesday (Jan 23), watch the videos:

- FTC – 2: 8.5, 8.6, 8.7
- Integration by substitution: 9.1, 9.2, 9.3, 9.4

Properties of the integral

Assume we know the following

$$\int_0^2 f(x)dx = 3, \qquad \int_0^4 f(x)dx = 9, \qquad \int_0^4 g(x)dx = 2.$$

Compute:

1. $\displaystyle\int_0^2 f(t)dt$
2. $\displaystyle\int_0^2 f(t)dx$
3. $\displaystyle\int_2^0 f(x)dx$
4. $\displaystyle\int_2^4 f(x)dx$
5. $\displaystyle\int_{-2}^0 f(x)dx$
6. $\displaystyle\int_0^4 [f(x) - 2g(x)]\ dx$

The Mean Value Theorem for integrals

Prove the following theorem.

Theorem

Let $a < b$. Let f be a continuous function on $[a, b]$.
There exists $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(t)dt$$

Hint: Bound $\int_a^b f(t)dt$ from above and from below using the maximum and minimum of f in $[a, b]$. (You need to use the EVT for that.) Then use the IVT.

Compute the integral of $f(x) = x^2$ on $[-1, 1]$

1. Why is $f(x) = x^2$ integrable on $[-1, 1]$?

2. Recall the result about Riemann sums.

3. Using your previous answer, compute $\int_{-1}^{1} f(x)dx$

Hint 1: You can construct a sequence of partitions of $[-1, 1]$ whose norm tends to 0 by breaking $[-1, 1]$ into n subintervals of the same length (and then to let n tends to ∞).

Hint 2: One easy way to tag the subintervals of a partition consists in picking the (right or left) endpoints.

Hint 3: Recall that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

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