Summation and the Σ notation

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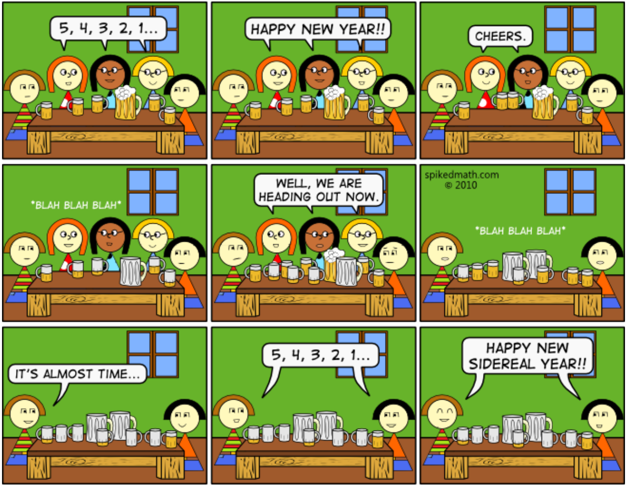
For next lecture

For Wednesday (Jan 9), watch the videos:

- Sup & Inf: 7.3, 7.4
- Integrable functions: 7.5, 7.6, 7.7



Warm-up: sums

Recall:

$$\sum_{i=p}^{n} a_i = a_p + a_{p+1} + a_{p+2} + \cdots + a_{n-1} + a_n$$

How many summands are there in the above sum?

Compute:

1. $\sum_{i=2}^{4} (2i+1)$
2. $\sum_{j=2}^{4} (2i+1)$
3. $\sum_{i=4}^{2} (2i+1)$
4. $\sum_{i=2}^{4} 2i + 1$

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Write these sums with Σ (the solution may not be unique)

1. $1^5 + 2^5 + 3^5 + 4^5 + \cdots + 100^5$
2. $\frac{2}{4^2} + \frac{2}{5^2} + \frac{2}{6^2} + \frac{2}{7^2} + \cdots + \frac{2}{N^2}$
3. $\cos 0 - \cos 1 + \cos 2 - \cos 3 + \cdots \pm \cos(N+1)$
4. $\frac{1}{1!} - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots + \frac{1}{81!}$
5. $\frac{x^2}{3!} + \frac{2x^3}{4!} + \frac{3x^4}{5!} + \frac{4x^5}{6!} + \cdots + \frac{999x^{1000}}{1001!}$

Double sums

Compute:

1. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{N} 1$
2. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} 1$
3. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} i$
4. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} k$
5. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} (ik)$

Useful formulae:

$$\sum_{j=1}^{N} j = \frac{N(N+1)}{2}, \quad \sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}, \quad \sum_{j=1}^{N} j^3 = \frac{N^2(N+1)^2}{4}$$

Telescopic sum

Compute the exact value of

$$\sum_{i=1}^{2{,}019}\left(\frac{1}{i}-\frac{1}{i+1}\right)$$

Hint: Write down the first few terms.

Compute the exact value of

$$\sum_{i=1}^{10{,}000}\frac{1}{i(i+1)}$$

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Re-writing sums

1. $$\sum_{i=1}^{100} \tan i - \sum_{i=1}^{50} \tan i = \sum_{???}^{???} ???$$

2. $$\sum_{i=1}^{N} (2i-1)^5 = \sum_{i=0}^{N-1} ???$$

3. $$\left(\sum_{k=1}^{N} x^k\right) + \left(\sum_{k=0}^{N} k\ x^{k+1}\right) = \left(\sum_{k=???}^{???} ???x^k\right) + ???$$