Indeterminate forms and L'Hôpital's Rule

UNIVERSITY OF TORONTO

November 28th, 2018

For next week

For Monday (Dec 3), watch the videos:

- Applied optimization: 6.1, 6.2

For Wednesday (Dec 5), watch the videos:

- Concavity: 6.11, 6.12
- Asymptotes: TBA

For Thursday (Dec 6), watch the videos:

- Curve sketching: TBA

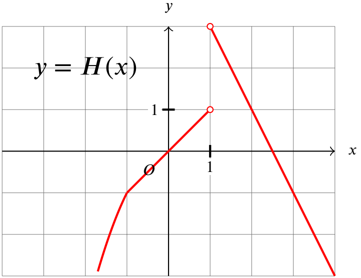
Computations

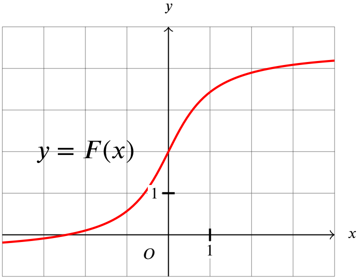
Compute:

1. $\lim_{x\to 2} \dfrac{x^2+2x-6}{x^2+3x-10}$
2. $\lim_{x\to 0} \dfrac{e^{2x^2}-\cos x}{x\sin x}$
3. $\lim_{x\to\infty} \dfrac{x^2}{e^x}$
4. $\lim_{x\to\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
5. $\lim_{x\to\infty} (\sin x)\left(e^{1/x}-1\right)$
6. $\lim_{x\to\infty} x\sin\dfrac{2}{x}$
7. $\lim_{x\to\infty} x\cos\dfrac{2}{x}$
8. $\lim_{x\to 1} \left[(\ln x)\tan\dfrac{\pi x}{2}\right]$

Compute:

1. $$\lim_{x \to 0} \frac{H(x)}{H(2+3x)-1}$$
2. $$\lim_{x \to 2} \frac{F^{-1}(x)}{x-2}$$





Construct a polynomial P such that

$$\lim_{x\to 1} \frac{P(x)}{e^x - e \cdot x} = \frac{1}{e}$$

Proving something is an indeterminate form

1. a Prove that $\forall c \in \mathbb{R}$, there exist functions f and g s.t.

$$\lim_{x\to 0} f(x) = 0, \quad \lim_{x\to 0} g(x) = 0, \quad \lim_{x\to 0} \frac{f(x)}{g(x)} = c$$

   b Find f and g such that

$$\lim_{x\to 0} f(x) = 0, \quad \lim_{x\to 0} g(x) = 0, \quad \lim_{x\to 0} \frac{f(x)}{g(x)} = +\infty$$

   c Same as above with $-\infty$.

2. Same for $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$.

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2. Same for $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$.

Infinity minus infinity

Compute:

1. $$\lim_{x\to 0}\left[\frac{\csc x}{x}-\frac{\cot x}{x}\right]$$
2. $$\lim_{x\to \infty}\left[\ln(x+2)-\ln(3x+4)\right]$$
3. $$\lim_{x\to 1}\left[\frac{2}{x^2-1}-\frac{1}{x-1}\right]$$
4. $$\lim_{x\to -\infty}\left[\sqrt{x^2+3x}-\sqrt{x^2-3x}\right]$$

Exponential indeterminate forms

Compute:

1. $\lim_{x\to 0} [1 + 2\sin(3x)]^{4\cot(5x)}$
2. $\lim_{x\to\infty} \left(\frac{x+2}{x-2}\right)^{3x}$
3. $\lim_{x\to 0^+} x^x$
4. $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$