MAT137Y1 – LEC0501

Calculus!

Indeterminate forms and L'Hôpital's Rule

November 28th, 2018

For next week

For Monday (Dec 3), watch the videos:

- Applied optimization: 6.1, 6.2

For Wednesday (Dec 5), watch the videos:

- Concavity: 6.11, 6.12
- Asymptotes: TBA

For Thursday (Dec 6), watch the videos:

- Curve sketching: TBA

Computations

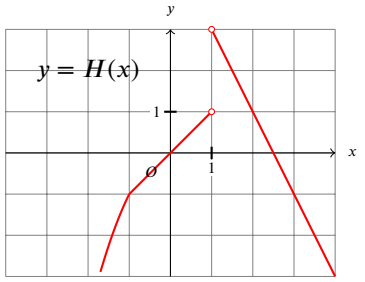
Compute:

1. $\lim\limits_{x\to 2} \dfrac{x^2+2x-6}{x^2+3x-10}$
2. $\lim\limits_{x\to 0} \dfrac{e^{2x^2}-\cos x}{x\sin x}$
3. $\lim\limits_{x\to \infty} \dfrac{x^2}{e^x}$
4. $\lim\limits_{x\to \infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
5. $\lim\limits_{x\to \infty} (\sin x)\left(e^{1/x}-1\right)$
6. $\lim\limits_{x\to \infty} x\sin\dfrac{2}{x}$
7. $\lim\limits_{x\to \infty} x\cos\dfrac{2}{x}$
8. $\lim\limits_{x\to 1} \left[(\ln x)\tan\dfrac{\pi x}{2}\right]$

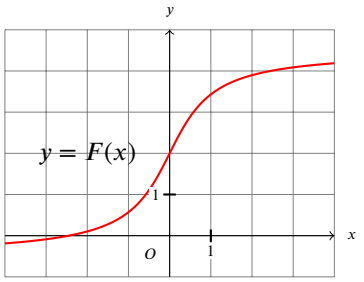
Limits from graphs

Compute:

1. $\lim\limits_{x\to 0} \dfrac{H(x)}{H(2+3x)-1}$



2. $\lim\limits_{x\to 2} \dfrac{F^{-1}(x)}{x-2}$



Backwards L'Hôpital

Construct a polynomial P such that

$$\lim_{x\to 1} \frac{P(x)}{e^x - e\cdot x} = \frac{1}{e}$$

Proving something is an indeterminate form

1. a Prove that $\forall c \in \mathbb{R}$, there exist functions f and g s.t.

$$\lim_{x\to 0} f(x) = 0, \quad \lim_{x\to 0} g(x) = 0, \quad \lim_{x\to 0} \frac{f(x)}{g(x)} = c$$

   b Find f and g such that

$$\lim_{x\to 0} f(x) = 0, \quad \lim_{x\to 0} g(x) = 0, \quad \lim_{x\to 0} \frac{f(x)}{g(x)} = +\infty$$

   c Same as above with $-\infty$.

2. Same for $\frac{\infty}{\infty}$, $0\cdot\infty$, and $\infty - \infty$.

Infinity minus infinity

Compute:

1. $\lim_{x\to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x}\right]$
2. $\lim_{x\to\infty} \left[\ln(x+2) - \ln(3x+4)\right]$
3. $\lim_{x\to 1} \left[\frac{2}{x^2-1} - \frac{1}{x-1}\right]$
4. $\lim_{x\to-\infty} \left[\sqrt{x^2+3x} - \sqrt{x^2-3x}\right]$

Exponential indeterminate forms

Compute:

1. $\lim_{x\to 0} [1+2\sin(3x)]^{4\cot(5x)}$
2. $\lim_{x\to\infty} \left(\frac{x+2}{x-2}\right)^{3x}$
3. $\lim_{x\to 0^+} x^x$
4. $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$