Inverse functions

University of Toronto

November 12th, 2018

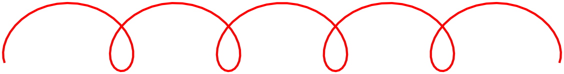
For next lecture

For Wednesday (Nov 14), watch the videos:

- One-to-one functions: 4.3, 4.4, 4.5
- Inverse trig functions: 4.6, 4.7, 4.8

Warm up

A worm is crawling accross the table.
The path of the worm looks something like this:

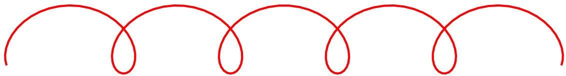


True or False?

The position of the worm in terms of time is a function.

Worm function

A worm is crawling accross the table.
For any time t, let $f(t)$ be the position of the worm.
This defines a function f.



1. What is the domain of f?
2. What is the codomain of f?
3. What is the range of f?
4. Does f admit an inverse?

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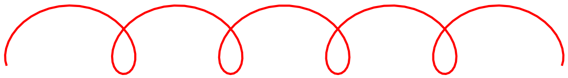

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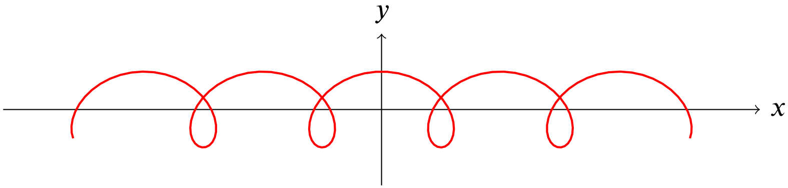
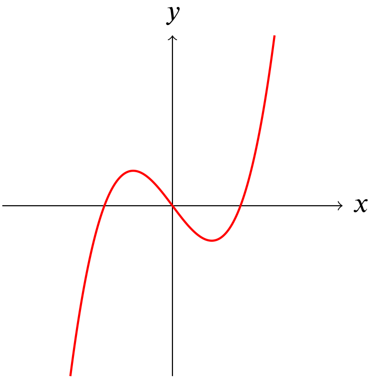
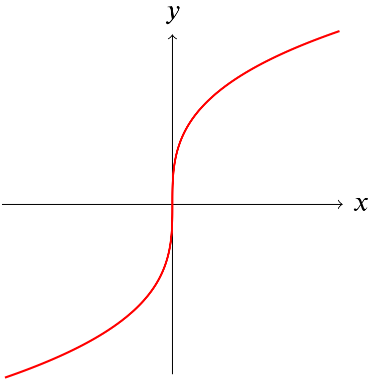
Is it the graph of a function?



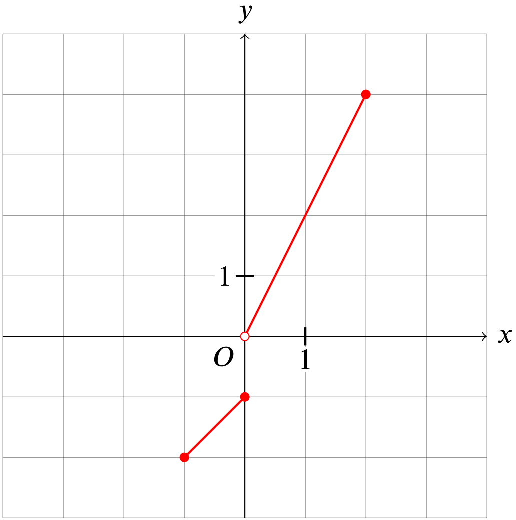
Do these functions admit an inverse?

If so, sketch the graph of the inverse.





Inverse function from a graph



Compute:

1. $f(2)$
2. $f(0)$
3. $f^{-1}(2)$
4. $f^{-1}(0)$
5. $f^{-1}(-1)$

Absolute value and inverses

Define the function $f : \mathbb{R} \to \mathbb{R}$ by

$$h(x) = x|x| + 1$$

1. Sketch the graph of h and explain briefly why it admits an inverse.
2. Compute $h^{-1}(-8)$.
3. Sketch the graph of h^{-1}.
4. Find an equation for $h^{-1}(x)$.
5. Verify that for every $t \in \mathbb{R}$, $h(h^{-1}(t)) = t$, and that for every $t \in \mathbb{R}$, $h^{-1}(h(t)) = t$.

Logarithmic differentiation: be careful!

Let $f(x) = xe^{\sin(x)}$.
We want to prove that $f'(x) = e^{\sin(x)} + x\cos(x)e^{\sin(x)}$ on $\mathbb{R}$.

What do you think about the following proof?

We have $\ln(f(x)) = \ln(xe^{\sin(x)}) = \ln(x) + \sin(x)$.
Hence, by differentiating w.r.t. x, we get

$$\frac{f'(x)}{f(x)} = \frac{1}{x} + \cos(x)$$

Thus

$$\begin{aligned} f'(x) &= f(x)\left(\frac{1}{x} + \cos(x)\right) \\ &= xe^{\sin(x)}\left(\frac{1}{x} + \cos(x)\right) \\ &= e^{\sin(x)} + x\cos(x)e^{\sin(x)} \end{aligned}$$