Differentiation rules (2)

UNIVERSITY OF TORONTO

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For next week

For Monday (Oct 29), watch the videos:

- Trig derivatives, implicit differentiation: 3.11, 3.12

For Wednesday (Oct 31), watch the videos:

- Derivatives of exponentials and logarithms: 3.13, (3.14), 3.15, 3.16, 3.17, 3.18
- Related rates: 3.19, 3.20

A different proof for the quotient rule

Assume we have already proven the product rule, the chain rule and we know the derivative of $p(x) = \frac{1}{x}$.

Obtain a formula for the derivative of $h(x) = \dfrac{f(x)}{g(x)}$.

A pesky function

Let $h(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

1. Calculate $h'(x)$ for any $x \neq 0$.
2. Is h continuous at 0?
3. Is h differentiable at 0? If so, compute $h'(0)$.
4. Is h' continuous at 0?

True or False

Let f be a function with domain $\mathbb{R}$ and $a \in \mathbb{R}$.
Assume that f is differentiable everywhere.

What can we conclude?

1. $f(a)$ is defined.
2. f is continuous at a.
3. $\lim_{x \to a} f(x)$ exists.
4. $f'(a)$ exists.
5. $\lim_{x \to a} f'(x)$ exists.
6. f' is continuous at a.

Higher order derivatives

Let $g(x) = \dfrac{1}{x^3}$.

Calculate the first few derivatives.

Make a conjecture for a formula of $g^{(n)}(x)$, the n-th derivative.
Prove it.

Computations

Compute the derivative of

1. $f(x) = \sqrt{2x^2 + x + 1}$
2. $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + 1}}}$

Iterated derivatives of $(f \times g)$

Assume f and g are functions that have all their derivatives.

Find formulas for the following derivatives:

1. $(fg)'(x)$
2. $(fg)''(x)$
3. $(fg)'''(x)$

Do these formulae seem familiar?
Make a conjecture for a formula for $(fg)^{(n)}$.

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Rate of change

Let f be a continuous function with domain $\mathbb{R}$.

True or False?

If the average rate of change of f between $x = 1$ and $x = 2$ is -3,
then f must be decreasing on $[1, 2]$.

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If, for every $1 \leq x_1 < x_2 \leq 2$,
the average rate of change of f between x_1 and x_2 is negative,
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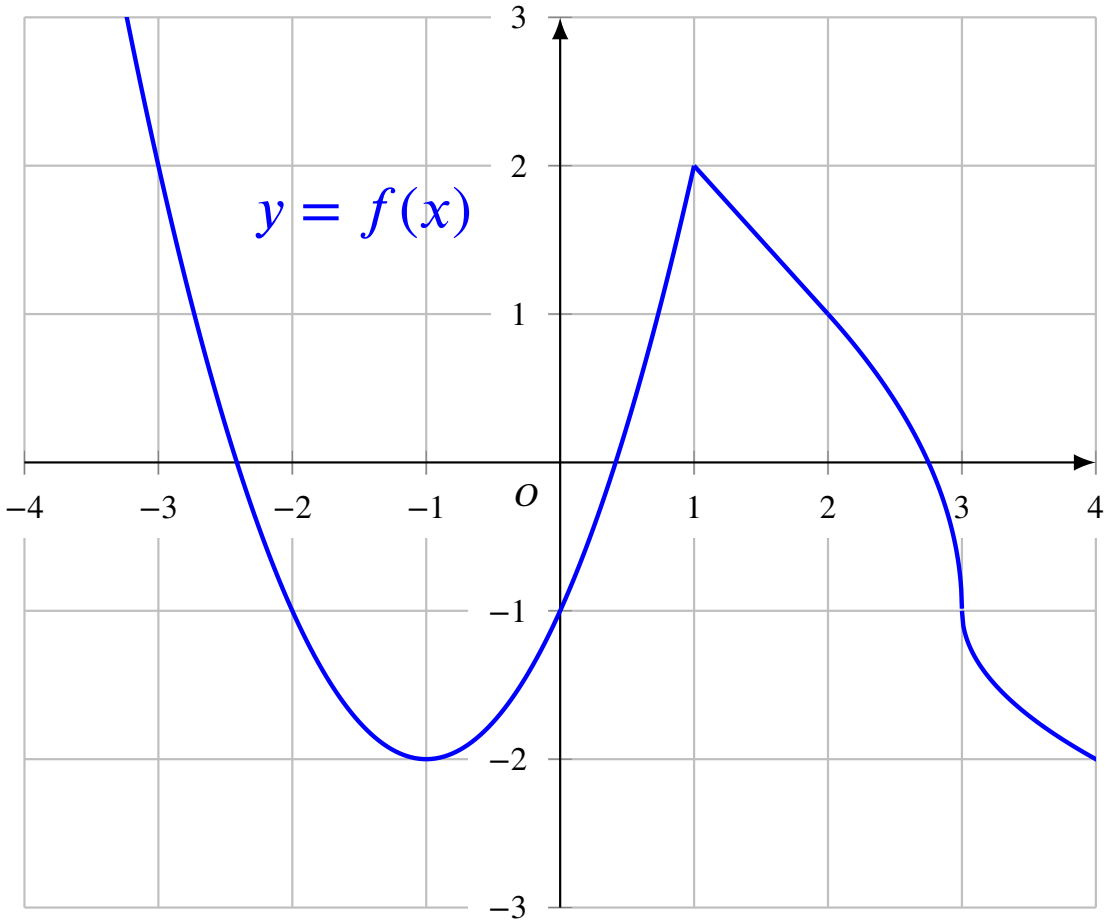
Tangent line from a graph

Below is the graph of a function f.
Write the equation of the line tangent to it at the point with x–coordinate -2.



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