Differentiation rules

October 22nd, 2018

For next lecture

For Wednesday (Oct 24), watch the video:

- The chain rule: 3.10

A continuity lemma

Write a formal proof for:

Lemma

Let $a \in \mathbb{R}$.
Let g be a function continuous at a.
If $g(a) \neq 0$ then $g(x) \neq 0$ for x close to a.

First, figure out what "$g(x) \neq 0$ for x close to a" means.

Hint:

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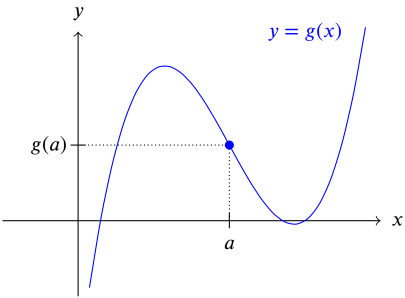

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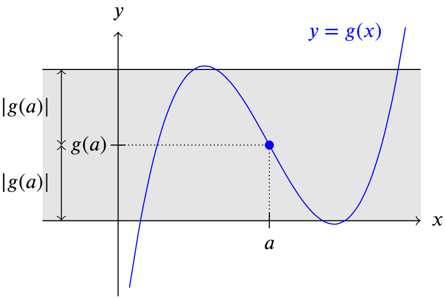

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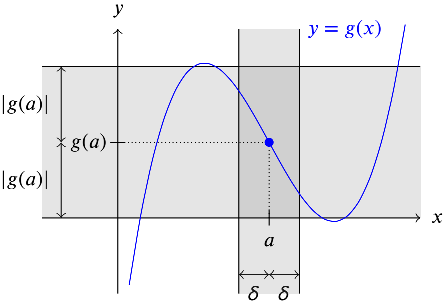

Write a formal proof for the quotient rule for derivatives

Theorem

- Let $a \in \mathbb{R}$.
- Let f and g be functions defined at and near a. Assume $g(a) \neq 0$.
- We define the function h by $h(x) = \dfrac{f(x)}{g(x)}$.

IF f and g are differentiable at a,

THEN h is differentiable at a, and

$$h'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{g(a)^2}.$$

Write a proof directly from the definition of derivative.
Hint: remember the proof of the product rule in video 3.6.

Computations

Compute the derivative of the following functions:

1. $f(x) = x^{100} + 3x^{30} - 2x^{15}$
2. $f(x) = \sqrt[3]{x} + 6$
3. $f(x) = \dfrac{4}{x^4}$
4. $f(x) = \sqrt{x}\,(1 + 2x)$
5. $f(x) = \dfrac{x^6 + 1}{x^3}$
6. $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$

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