MAT137Y1 – LEC0501

Calculus!

Definition of a derivative

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For next week

For Monday (Oct 22), watch the videos:

- Differentiation rules: 3.4, 3.5, 3.6, 3.7, 3.8, 3.9

For Wednesday (Oct 24), watch the videos:

- The chain rule: 3.10

Absolute value and tangent lines

1. Sketch the graph $y = |x|$.
2. At (0,0) the graph of $y = |x|$...
   1. ... has one tangent line: $y = 0$
   2. ... has one tangent line: $x = 0$
   3. ... has two tangent lines $y = x$ and $y = -x$
   4. ... has no tangent line
3. Give the domain of differentiability of $f(x) = |x|$ and an explicit description of $f'(x)$ whenever it is defined.

Absolute value and derivatives

Let $h(x) = x|x|$. What is $h'(0)$?

1. It does not exist because $|x|$ is not differentiable at 0.
2. It does not exist because the right- and left-limits, when computing the derivative, are different.
3. It does not exist because it has a corner.
4. It is 0.
5. It is 1.

Compute a derivative from the definition

Let $f(x) = \sqrt{x}$ defined on $[0, +\infty)$.

Determine the domain of the derivative of f.

Give an explicit description of $f'(x)$ whenever f is differentiable at x.

Are the following functions differentiable at...?

If yes, give the derivative.

1. $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ at $x_0 = 0$.

2. $g(x) = \begin{cases} \frac{|x|\sqrt{x^2-2x+1}}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$ at $x_0 = 0$ and at $x_1 = 1$.

Estimation

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Differentiability $\implies$ Continuity

Prove the following claim:

Claim

If f is differentiable at x then f is continuous at x.

Is the converse true?