IVT AND EVT

UNIVERSITY OF TORONTO

October 17th, 2018

For next week

For Monday (Oct 22), watch the videos:

- Differentiation rules: 3.4, 3.5, 3.6, 3.7, 3.8, 3.9

For Wednesday (Oct 24), watch the videos:

- The chain rule: 3.10

Definition of maximum

Let f be a function with domain I.
Which one (or ones) of the following is (or are) a definition of "f has a maximum on I"?

1. $\forall x \in I,\ \exists C \in \mathbb{R},\ f(x) \leq C$
2. $\exists C \in I,\ \forall x \in I,\ f(x) \leq C$
3. $\exists C \in \mathbb{R},\ \forall x \in I,\ f(x) \leq C$
4. $\exists C \in \mathbb{R},\ \forall x \in I,\ f(x) < C$
5. $\exists a \in I$ s.t. $\forall x \in I,\ f(x) \leq f(a)$
6. $\exists a \in I$ s.t. $\forall x \in I,\ f(x) < f(a)$

The challenge from the video

Find a function with domain $(0, 1]$ such that:

1. f is continuous on $(0, 1]$
2. f has no max on $(0, 1]$
3. f has no min on $(0, 1]$

The challenge from the video

Find a function with domain $(0, 1]$ such that:

1. f is continuous on $(0, 1]$
2. f has no max on $(0, 1]$
3. f has no min on $(0, 1]$

The assumptions of the EVT are that:

- f is **continuous**
- on an interval $[a, b]$ (**closed** at both sides).

Extrema

In each of the following cases, does the function f have a maximum and a minimum on the interval I?

1. $f(x) = x^2, \quad I = (-1, 1)$

2. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [2, 6]$

3. $f(x) = \begin{cases} \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3 & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}, \quad I = [-2, 2]$

Existence

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

Prove the following claim:

Claim

Let f be a function continuous on $\mathbb{R}$.
If for all $x \in \mathbb{R}$, $f(x)^2 = 1$,
then either $f = 1$ or $f = -1$.