Limit laws

October 3rd, 2018

For next week

For Wednesday (Oct 10), watch the videos:

- Continuity: 2.14, 2.15, 2.16, 2.17, 2.18.

Is this proof correct?

Claim:

$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x \in \mathbb{R},\ 0 < |x| < \delta \implies |x^3 + x^2| < \varepsilon$$

Proof.

- Let $\varepsilon > 0$.
- Take $\delta = \sqrt{\frac{\varepsilon}{|x+1|}}$.
- Let $x \in \mathbb{R}$. Assume $0 < |x| < \delta$. Then

$$|x^3 + x^2| = x^2|x+1| < \delta^2|x+1| = \frac{\varepsilon}{|x+1|}|x+1| = \varepsilon.$$

- I have proved that $|x^3 + x^2| < \varepsilon$

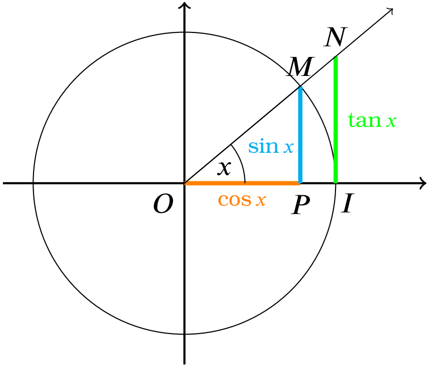
□

Compute a limit using geometry

Compute the following limits:

1. $\lim_{x\to 0} \sin(x)$
2. $\lim_{x\to 0} \cos(x)$
3. $\lim_{x\to 0} \frac{\sin(x)}{x}$

Hint: first, compute the limit at 0^+ using the unit circle.



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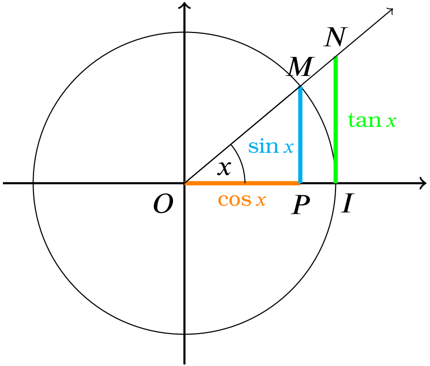

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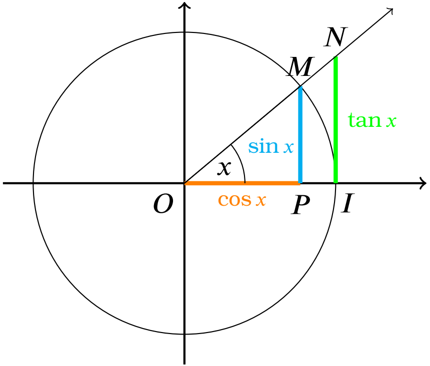

A question from last year's test (limit laws)

The only thing we know about the function g is that

$$\lim_{x\to 0} \frac{g(x)}{x^2} = 2.$$

Compute the following limits (or explain that they do not exist):

1. $\lim_{x\to 0} \frac{g(x)}{x}$
2. $\lim_{x\to 0} \frac{g(x)}{x^4}$
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1. $\lim_{x\to\infty} \lfloor x \rfloor$
2. $\lim_{x\to 0} x \left\lfloor \frac{1}{x} \right\rfloor$
3. $\lim_{x\to 0} x^2 \left\lfloor \frac{1}{x} \right\rfloor$

Remember that, for $x \in \mathbb{R}$ and $m \in \mathbb{R}$,

$$\begin{aligned} \lfloor x \rfloor = m &\Leftrightarrow \ m \le x < m+1 \\ &\Leftrightarrow x - 1 < m \le x \end{aligned}$$

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Indeterminate form

Let f and g be functions defined near 0.
Assume $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$.

What can we conclude about $\lim_{x\to 0} \frac{f(x)}{g(x)}$?

1. The limit is 1.
2. The limit is 0.
3. The limit is ∞.
4. The limit does not exist.
5. We do not have enough information to decide.

Proof of non-existence

Goal

We want to prove that

$$\lim_{x \to 0^+} \frac{1}{x} \quad \text{does not exist,} \tag{1}$$

directly from the definition.

1. Write down formally the statement (1).
2. Write down the structure of the formal proof should be, without filling the details.
3. Rough work.
4. Write down a complete formal proof.

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Proof of a limit law

Theorem

Let f and g be functions with domain $\mathbb{R}$, except possibly a.
IF

- $\lim_{x \to a} f(x) = L \in \mathbb{R}$, and
- $\lim_{x \to a} g(x) = M \in \mathbb{R}$.

THEN

$$\lim_{x \to a} \left(f(x) + g(x) \right) = L + M$$

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