MAT137Y1 – LEC0501

Calculus!

PROOFS USING THE DEFINITION OF LIMITS

UNIVERSITY OF TORONTO

October 1st, 2018

For next lecture

For Wednesday (Oct 3), watch the videos:

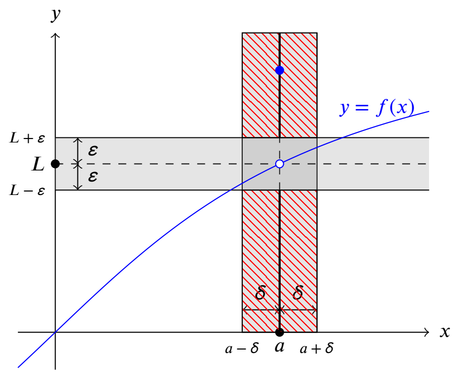
- Limit laws: 2.10, 2.11, 2.12, 2.13.

Graphical interpretation

Remember that $\lim_{x\to a} f(x) = L$ means

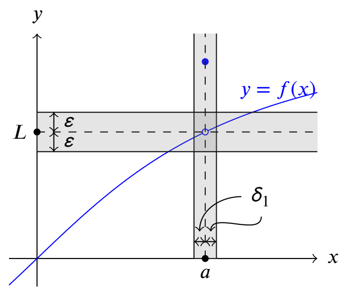
$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x \in \mathbb{R},\ \big(0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon\big)$$

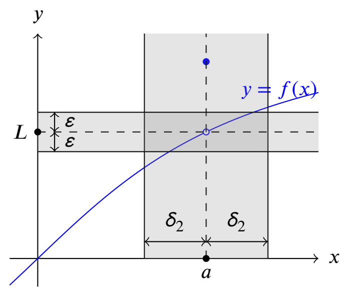
What are a, L, δ, ε in the figure below?

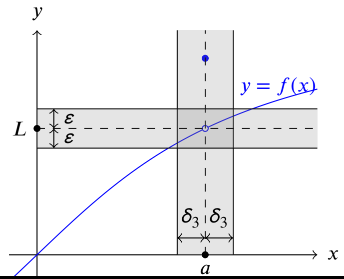


Graphical interpretation (2)

Assume that ε is given. Which of the following δ_i works?







$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x \in \mathbb{R},$$
$$\big(0 < |x-a| < \delta \implies |f(x) - L| < \varepsilon\big)$$

Choose δ for a given ε

1. Find a $\delta > 0$ such that

$$\forall x \in \mathbb{R},\ |x-3| < \delta \implies |5x-15| < 1$$

2. Find all $\delta > 0$ such that

$$\forall x \in \mathbb{R},\ |x-3| < \delta \implies |5x-15| < 1$$

3. Let $\varepsilon > 0$. Find all $\delta > 0$ such that

$$\forall x \in \mathbb{R},\ |x-3| < \delta \implies |5x-15| < \varepsilon$$

4. Prove $\lim_{x \to 3} 5x = 15$.

Your first $\varepsilon - \delta$ proof

Goal

We want to prove that

$$\lim_{x \to 1} \left(x^2 + x\right) = 2 \qquad (1)$$

directly from the definition.

1. Write down the formal definition of the statement (1).
2. Write down what the structure of the formal proof should be, without filling the details.
3. Write down a complete formal proof.