MAT137Y1 – LEC0501

Calculus!

Absolute value, distance & inequalities

September 24th, 2018

For next lecture

For Wednesday (Sep 26), watch the videos:

- Limits: 2.1, 2.2, 2.3, 2.5, 2.6

Properties of inequalities

Let $a, b, c \in \mathbb{R}$.
Assume $a < b$. Which of the followings are true, and why?

1. $a + c < b + c$
2. $a - c < b - c$
3. $ac < bc$
4. $a^2 < b^2$
5. $\frac{1}{a} < \frac{1}{b}$

Properties of absolute value

Let $x, y \in \mathbb{R}$.
Which of the followings are true, and why?

1. $|xy| = |x||y|$
2. $|x + y| = |x| + |y|$

Sets defined by distance

Let $a, \delta \in \mathbb{R}$.
Describe the following sets in terms of intervals.

1. $A = \{x \in \mathbb{R},\ |x| \leq \delta\}$
2. $B = \{x \in \mathbb{R},\ |x| > \delta\}$
3. $C = \{x \in \mathbb{R},\ |x - a| > \delta\}$
4. $D = \{x \in \mathbb{R},\ 0 < |x - a| < \delta\}$
5. $E = \{x \in \mathbb{R},\ |x - 1| = |x + 1|\}$

A new function

For $x, y \in \mathbb{R}$, we set

$$f(x, y) = \frac{x + y - |x - y|}{2}$$

Find a simpler description for f.

Implications

Find *all* values of A, B, and C that make the following implications true.

1. $\forall x \in \mathbb{R},\ |x - 3| < 1 \implies |2x - 6| < A$
2. $\forall x \in \mathbb{R},\ |x - 3| < B \implies |2x - 6| < 1$
3. $\forall x \in \mathbb{R},\ |x - 3| < 1 \implies |x + 5| < C$

Triangle inequality[1]

Prove the following statements:

1. $\forall x, y \in \mathbb{R},\ |x + y| \leq |x| + |y|$
2. $\forall x, y \in \mathbb{R},\ |x - y| \leq |x| + |y|$
3. $\forall x, y \in \mathbb{R},\ \big||x| - |y|\big| \leq |x - y|$

[1] This slide was not used during the lecture. You can use it to train yourself.