CONDITIONAL STATEMENTS

UNIVERSITY OF
TORONTO

September 17th, 2018

For next lecture

For Wednesday (Sep 19), watch the videos:

- Proofs and definitions: 1.10, 1.11, 1.12, 1.13, 1.14, 1.15

The Eiffel Tower is in Toronto

Is the following statement true or false?

If the CN Tower is in France then the Eiffel Tower is in Toronto.

Write the negation of the above statement.

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Mnemonic device: "from falsehood, anything follows".

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True or False?

1 If $0 > 1$ then $1 > 0$.

2 If $0 > 1$ then $0 > 1$.

3 If $0 < 1$ then $1 > 0$.

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5 $\forall x \in \mathbb{R}, \; [x \in \mathbb{N} \Rightarrow x \in \mathbb{Z}]$.

6 $\forall x \in \mathbb{R}, \; [x \in \mathbb{Z} \Rightarrow x \in \mathbb{N}]$.

7 $\forall a, b \in \mathbb{R}, \; \left[(a + b)^2 = a^2 + b^2 \Leftrightarrow a = 0 \text{ or } b = 0\right]$.

8 $\forall a, b \in \mathbb{R}, \; \left[a = b \Leftrightarrow a^2 = b^2\right]$.

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Increasing function

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3 Find a function which is not (strictly) increasing and not (strictly) decreasing.

Cards[1]

Four cards lie on the table in front of you.

You know that each card has a letter on one side and a natural number on the other.

At the moment, you can read the symbols E, P, 3 and 8 on the sides that are up.

I tell you:

> *"If a card has a vowel on one side,*
> *then it has an odd number on the other side."*

Which cards do you need to turn over in order to verify whether I am telling the truth or not?

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If $2 < x < 4$ then $1 < f(x) < 2$.

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Jean-Baptiste Campesato　　　**MAT137Y1 – LEC0501 – Calculus! – Sep 17, 2018**　　　**7**

A last one on conditionals

The following statement is false:

If today is my birthday then $2 = 1$.

What can you guess?