SETS AND QUANTIFIERS

UNIVERSITY OF
TORONTO

September 12th, 2018

For next week

For Monday (Sep 17), watch the videos:

- Conditionals: 1.7, 1.8, 1.9

For Wednesday (Sep 19), watch the videos:

- Proofs and definitions: 1.10, 1.11, 1.12, 1.13, 1.14, 1.15

True or False

1 $\frac{2}{3} \in \mathbb{N}$

2 $3 \in (1, 6]$

3 $[1, 3) \subseteq (1, 6)$

4 $(1, 2) \subseteq (1, 6)$

5 $\mathbb{Q} \subseteq \mathbb{R}$

6 $\mathbb{Z} \subseteq \mathbb{N}$

7 $\varnothing \subseteq \{1, 2, 4, 8, 16\}$

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Intervals

Write the following sets in a simpler manner.

1. $[2, 4] \cup (2, 5)$
2. $[2, 4] \cap (2, 5)$
3. $[\pi, e)$
4. $[0, 0]$
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Intervals, again…

Which of the following sets are intervals?

1 The set of real numbers greater than or equal to $\sqrt{2}$.

2 The set of real numbers less than 1 or greater than $\sqrt{2}$.

3 The set of real numbers greater than 1 and less than $\sqrt{2}$.

4 The set of real numbers less than 1 and greater than $\sqrt{2}$.

⚠ The words "greater than" and "less than" are not *inclusive*.[1]

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[1] However, it may not be the case in other languages. Mathematical symbols allow us to avoid the imprecisions of natural languages: there is no confusion between $>$ and $\geq$.
An example of possible confusion in English is: "increasing" versus "non-decreasing" function.

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Similar sets

What are the following sets?

1. $\{x \in \mathbb{R} \ : \ x^2 < 6\}$
2. $\{x \in \mathbb{Z} \ : \ x^2 < 6\}$
3. $\{x \in \mathbb{N} \ : \ x^2 < 6\}$

⚠ Be careful with the definition of $\mathbb{N}$.

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Is the following statement true or false? Prove it!

$$(A \cup B) \cap C = A \cup (B \cap C)$$

Write the negations of the following statements without using any negative words ("no", "not", "none", etc…):

1 *"Every page in this book has an odd number of words."*

2 *"Every page in this book contains at least one word whose first and last letters both come alphabetically before M."*

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Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Write each of the following sentences using mathematical symbols.

1. f doesn't vanish (ie. it never takes the value 0).

2. f is the zero function (ie. it is always 0).

3. f is not the zero function.

4. f is bounded from above.

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Quantifiers and functions

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Quantifiers

Are the following statements true?

1. There is a pink rhinoceros in the classroom.
2. Every rhinoceros in the classroom is pink.

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⚠ A statement of the form $\exists x \in \varnothing, \ P(x)$ is always false.

⚠ A statement of the form $\forall x \in \varnothing, \ P(x)$ is *vacuously true*.

Write the negations of the following sentences:

1 My favourite real number is greater than π or less than or equal to $\sqrt{2}$.

2 There is a country in the European Union with fewer than 1000 inhabitants.

3 Every student attending MAT137 has a mobile phone.

4 Every instructor of MAT137 with green eyes will win at the lottery and will retire before 50.

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[2]This slide was not used during the class. You can use it to train yourself.

Jean-Baptiste Campesato   **MAT137Y1 – LEC0501 – Calculus! – Sep 12, 2018**   **11/11**

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