

SETS AND QUANTIFIERS

September 12th, 2018

True or False

- 1 $\frac{2}{3} \in \mathbb{N}$
- 2 $3 \in (1, 6]$
- 3 $[1, 3) \subseteq (1, 6)$
- 4 $(1, 2) \subseteq (1, 6)$
- 5 $\mathbb{Q} \subseteq \mathbb{R}$
- 6 $\mathbb{Z} \subseteq \mathbb{N}$
- 7 $\emptyset \subseteq \{1, 2, 4, 8, 16\}$

For next week

For Monday (Sep 17), watch the videos:

- Conditionals: 1.7, 1.8, 1.9

For Wednesday (Sep 19), watch the videos:

- Proofs and definitions: 1.10, 1.11, 1.12, 1.13, 1.14, 1.15

Intervals

Write the following sets in a simpler manner.

- 1 $[2, 4] \cup (2, 5)$
- 2 $[2, 4] \cap (2, 5)$
- 3 $[\pi, e)$
- 4 $[0, 0]$
- 5 $(0, 0)$
- 6 $[0, 0)$

Intervals, again...

Which of the following sets are intervals?

- 1 The set of real numbers greater than or equal to $\sqrt{2}$.
- 2 The set of real numbers less than 1 or greater than $\sqrt{2}$.
- 3 The set of real numbers greater than 1 and less than $\sqrt{2}$.
- 4 The set of real numbers less than 1 and greater than $\sqrt{2}$.

⚠ The words “greater than” and “less than” are not *inclusive*.¹

¹However, it may not be the case in other languages. Mathematical symbols allow us to avoid the imprecisions of natural languages: there is no confusion between $>$ and \geq .

An example of possible confusion in English is: “increasing” versus “non-decreasing” function.

True or False

Is the following statement true or false? Prove it!

$$(A \cup B) \cap C = A \cup (B \cap C)$$

Similar sets

What are the following sets?

- 1 $\{x \in \mathbb{R} : x^2 < 6\}$
- 2 $\{x \in \mathbb{Z} : x^2 < 6\}$
- 3 $\{x \in \mathbb{N} : x^2 < 6\}$

⚠ Be careful with the definition of \mathbb{N} .

Negation

Write the negations of the following statements without using any negative words (“no”, “not”, “none”, etc...):

- 1 “Every page in this book has an odd number of words.”
- 2 “Every page in this book contains at least one word whose first and last letters both come alphabetically before M.”

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function. Write each of the following sentences using mathematical symbols.

- 1 f doesn't vanish (ie. it never takes the value 0).
- 2 f is the zero function (ie. it is always 0).
- 3 f is not the zero function.
- 4 f is bounded from above.

Are the following statements true?

- 1 There is a pink rhinoceros in the classroom.
- 2 Every rhinoceros in the classroom is pink.

Negation²

Write the negations of the following sentences:

- 1 My favourite real number is greater than π or less than or equal to $\sqrt{2}$.
- 2 There is a country in the European Union with fewer than 1000 inhabitants.
- 3 Every student attending MAT137 has a mobile phone.
- 4 Every instructor of MAT137 with green eyes will win at the lottery and will retire before 50.

²This slide was not used during the class. You can use it to train yourself.