

# Calculus!

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WELCOME TO MAT137!

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UNIVERSITY OF  
TORONTO

September 10<sup>th</sup>, 2018

# Information about this section:

Jean-Baptiste (JB) Campesato



`campesat@math.toronto.edu`



Please start the subject with “MAT137:”

Room MP202. Schedule:



- Monday, 4pm to 5pm
- Wednesday, 4pm to 6pm

Website for this section:



<http://www.math.toronto.edu/campesat/mat137.html>  
(no need to copy the slides in your notes, they are online)

# Resources:



MAT137 website: <http://uoft.me/MAT137>

**Visit this webpage regularly**, it contains:

- The outline of the course (read it!)
- The videos!
- Announcements
- Piazza: an online forum
- Timetable for the office hours  
(you can attend any office hours, not only mine)
- And more...



Precalculus resources:

<http://uoft.me/precalc>



Enrol in a tutorial!

# Goals of the course

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- **Calculus:** understanding of the basic concepts and results of calculus.
- **Mathematical logic:** being able to understand a mathematical proof and being able to write (even a short) proof.
- **Problem solving:** improving your problem solving skills to react when you face a problem which is new to you.

## Organization:

Before each lecture: watch the indicated videos.

During each lecture, I will give you the list of the videos you will have to watch before the next lecture.

You may also find this list on the webpage of the section.

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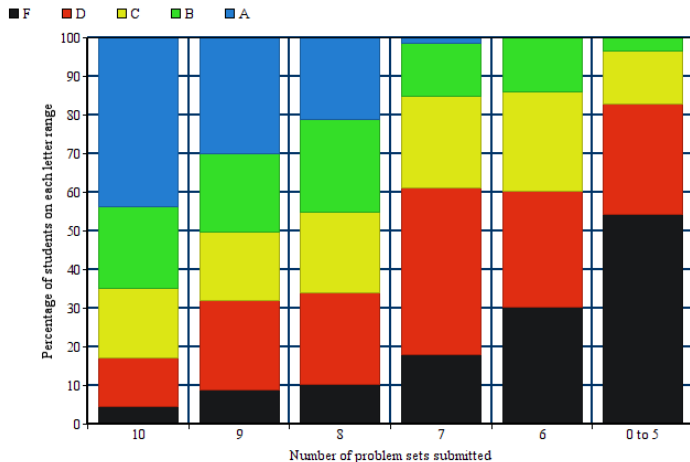
For Wednesday (Sep 12), watch the videos:

- Sets: 1.1, 1.2
- Quantifiers: 1.3, 1.4, 1.5, 1.6

# How did the students do last year?

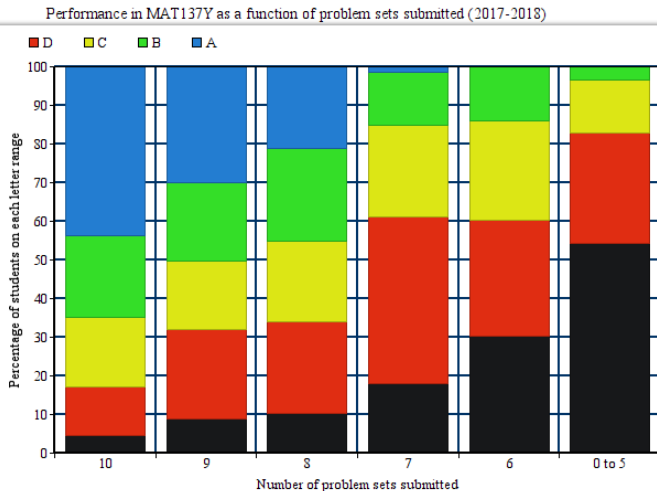
It depends on how many problem sets they submitted:

Performance in MAT137Y as a function of problem sets submitted (2017-2018)



# How did the students do last year?

It depends on how many problem sets they submitted:

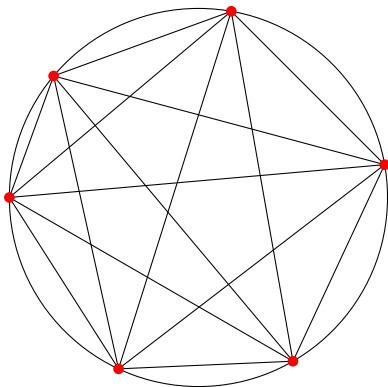


**Advice:** work on the problem sets, *practice makes perfect.*

# A warm up problem

Put  $n$  points on a circle and then join each pair of these points by a line segment.<sup>1</sup>

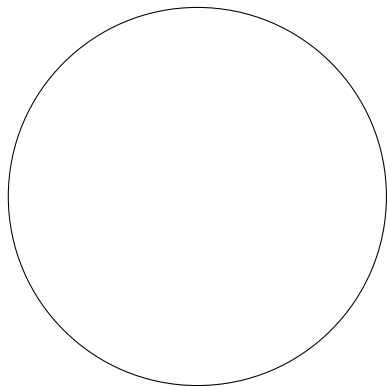
**In how many regions is the circle divided?**



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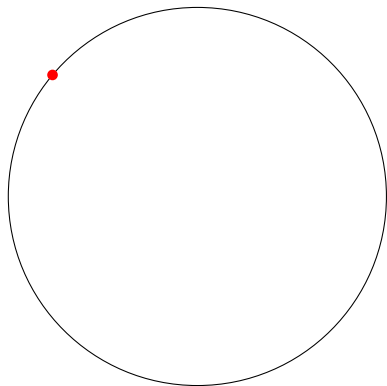
<sup>1</sup>We assume that inside the circle, three of these line segments never intersect at the same point.

# A warm up problem: how many regions do we obtain?



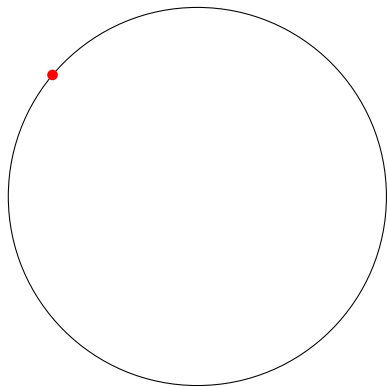
# of points	# of regions
1	
2	
3	
4	
5	
6	

# A warm up problem: how many regions do we obtain?



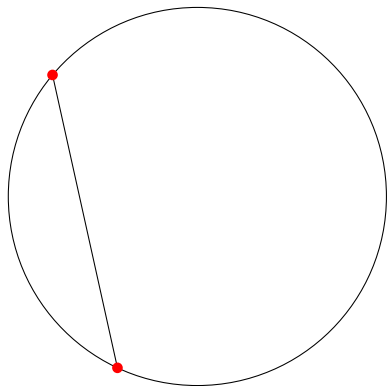
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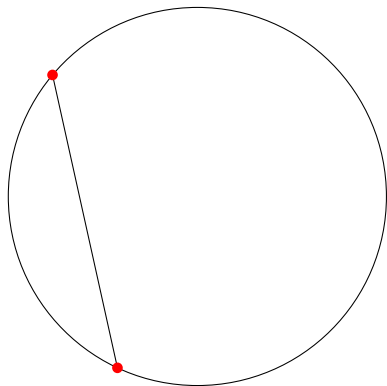
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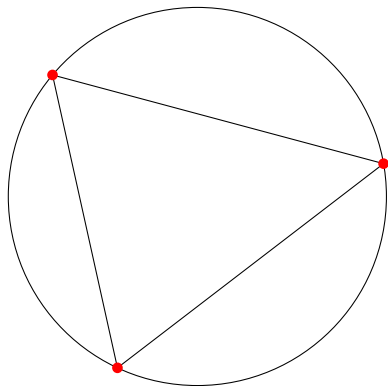


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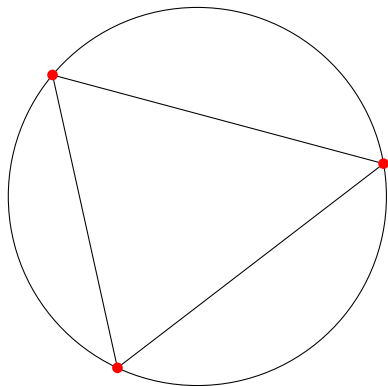
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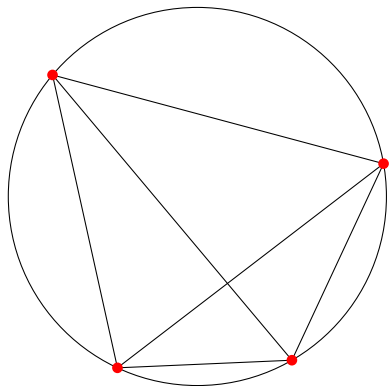
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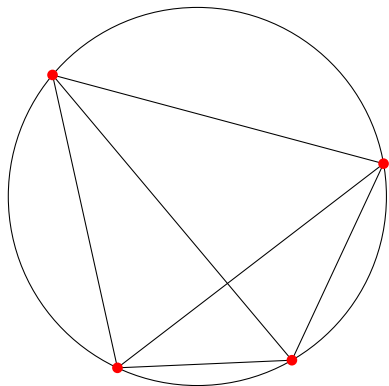
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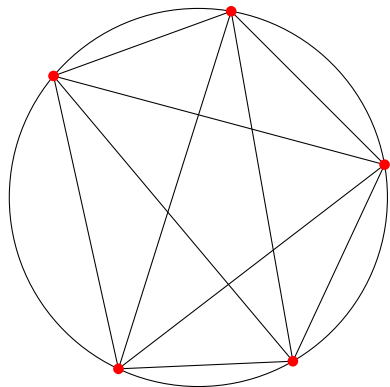
# A warm up problem: how many regions do we obtain?



# of points	# of regions
1	1
2	2
3	4
4	8
5	
6	

Any idea for a general formula?

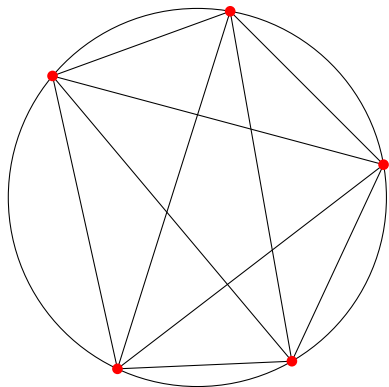
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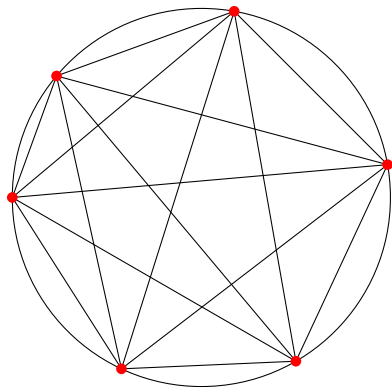
# A warm up problem: how many regions do we obtain?



# of points	# of regions
1	1
2	2
3	4
4	8
5	16
6	

Any idea for a general formula?  
Does it still hold?

# A warm up problem: how many regions do we obtain?

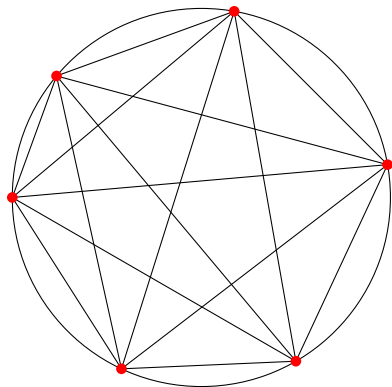


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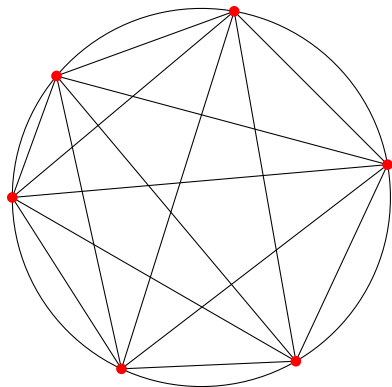
# A warm up problem: how many regions do we obtain?



# of points	# of regions
1	1
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5	16
6	32

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# A warm up problem: how many regions do we obtain?



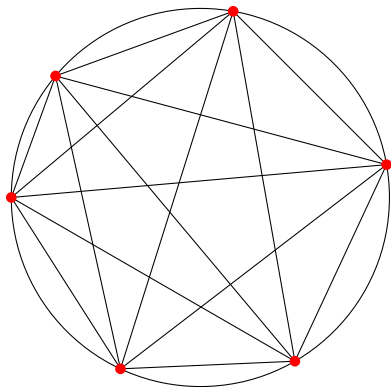
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Any idea for a general formula?

Does it still hold?

Hmmm...Let's count again! Just to be sure...

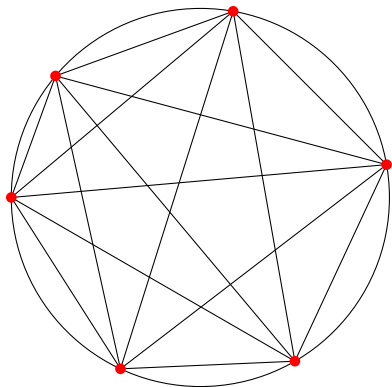
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6	<del>32</del> 31

Actually, the formula is not  $2^{n-1}$  but

$$\binom{n-1}{0} + \binom{n-1}{1} + \binom{n-1}{2} + \binom{n-1}{3} + \binom{n-1}{4} = \frac{n}{24}(n^3 - 6n^2 + 23n - 18) + 1$$

# A warm up problem: how many regions do we obtain?

## Conclusion

Examples are great to help you to understand new things, to find new ideas, to guess a formula...

But they are not enough!

We need **proofs** to be sure that a result is true in general.

# A warm up problem: how many regions do we obtain?

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But they are not enough!

We need **proofs** to be sure that a result is true in general.

⚠ Warning: I didn't say that examples are useless.

They are very important. I use them a lot when I want to understand a mathematical notion which is new to me.

When you face a new mathematical result, I *strongly* encourage you to “challenge” it on examples.

It may help you to better understand it!

## Another warm up problem

Which of the following statements are equivalent to this one:

“No two students in this class are not on fire.”

- 1 “For any pair of students in this class, at least one of them is on fire.”
- 2 “At least two students in this class are on fire.”
- 3 “All student in this class, except at most one, are on fire.”
- 4 “At least two students in this class are not on fire.”
- 5 “If I choose two students in this class and one of them is not on fire, then the other one is on fire.”

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 Just in case, notice the location of the closest extinguisher... 😊