Trigonometry Test Solutions.

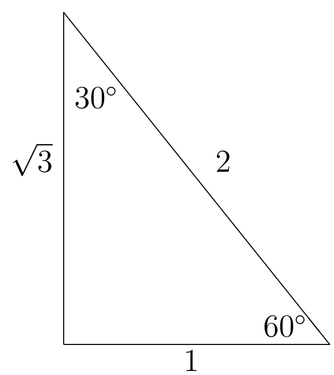
Answers to the Test:
1. F 2. T 3. T 4. C 5. B 6. A 7. D
Solutions and Comments:

1. For all x, $\sin(2x) = 2\sin x$; this is False.

   For instance, if $x = \dfrac{\pi}{2}$, then $2\sin x = 2(1) = 2$, but $\sin(2x) = \sin\pi = 0$. What is true is that
$$\sin(2x) = 2\sin x\cos x.$$

2. $\tan\dfrac{\pi}{3} = \sqrt{3}$; this is True.

   Everybody should know the trig ratios of the $\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{\pi}{2}$ (or $30°, 60°, 90°$) triangle, at the right. So
$$\tan\frac{\pi}{3} = \tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}.$$



3. If θ is in the second quadrant, then $\sqrt{1-\sin^2\theta} = -\cos\theta$; this is True.

   Simplify:
$$\sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta| = -\cos\theta,$$
   since $\cos\theta < 0$ in the second quadrant.

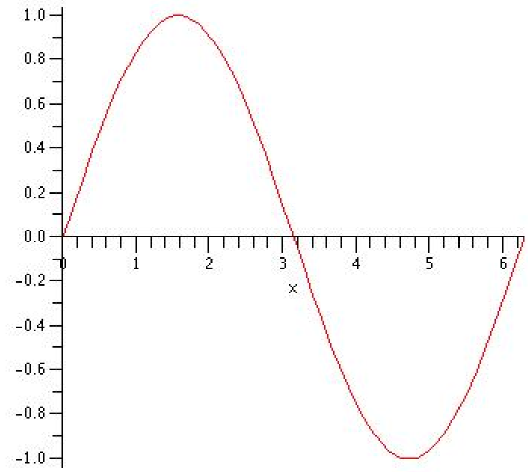
4. The number of solutions to the equation
$$2\sin^2 x - \sin x - 1 = 0$$
   in the interval $0 \le x \le 2\pi$ is 3.

   Factor and solve:

$$
\begin{aligned}
& 2\sin^2 x - \sin x - 1 = 0 \\
\Rightarrow\ & (2\sin x + 1)(\sin x - 1) = 0 \\
\Rightarrow\ & 2\sin x + 1 = 0 \text{ or } \sin x - 1 = 0 \\
\Rightarrow\ & \sin x = -\frac{1}{2} \text{ or } \sin x = 1
\end{aligned}
$$

From the graph of $\sin x$ to the right, you can see there are three solutions if $0 \leq x \leq 2\pi$. Aside: the solutions are

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}.$$



5. If $\sin x = \dfrac{3}{4}$ and $\cos x < 0$, then the exact value of $\tan x = -\dfrac{3}{\sqrt{7}}$.

   Use the basic trig identity, $\sin^2 x + \cos^2 x = 1$, to find $\cos x$:

$$\cos x = -\sqrt{1 - \sin^2 x} = -\sqrt{1 - \left(\frac{3}{4}\right)^2} = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}.$$

   Then

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = -\frac{3}{\sqrt{7}}.$$

6. The radian measure of $45°$ is

$$45\left(\frac{\pi}{180}\right) = \frac{\pi}{4}.$$

7. If a right triangle has sides of length $9, 40$ and 41 and α is the angle between the sides of length 9 and 41, then

$$\sin \alpha = \frac{40}{41}.$$

   From the triangle to the right, you can see that

$$\sin \alpha = \frac{40}{41}.$$