MAT186H1F CALCULUS I: Syllabus for Fall 2019 as of September 1, 2019

2019-2020 Calendar Description:

> Topics include: limits and continuity; differentiation; applications of the derivative—related rates problems, curve sketching, optimization problems, L'Hopital's rule; definite and indefinite integrals; the Fundamental Theorem of Calculus; applications of integration in geometry, mechanics and other engineering problems.

A lot of the material will be review of high school material, but not *all* of it; you must pay attention and assimilate the new material along with the old. We do assume that everything about integration is new to you. To intersperse the new and the old, we will introduce some of the material about integration as soon as possible.

Section Instructors: by now you should be scheduled into one of the following Sections:

LEC0101 D. Burbulla	LEC0102 S. Cohen	LEC0103 J. Arbunich	LEC0104 J. Ko
LEC0105 D. Burbulla	LEC0106 J. Han	LEC0107 L. Shorser	LEC0108 M. Breeling

Engineering Timetable: you should check the current timetable for Engineering math courses periodically, especially near the beginning of term since changes may occur.

Textbooks: the references for the course are three on-line open-source free textbooks:

1. Calculus, Volume 1, by Strang and Herman (C1)
2. Active Calculus 2.1 (C2)
3. APEX Calculus I, U of T version (C3)

C3 was our principal reference last year. This year we will rely mainly on C1; it is also the lead-in to the book(s) used in MAT187H1S Calculus II: Calculus, Volume 2 and Calculus, Volume 3. C2, in its html format, is notable for its WeBWorK interactive exercises, so is a good source of some practice problems. C3 will serve as an alternate reference, particularly when C1 doesn't cover a topic in enough depth. Last year there was another source that some students liked: Calculus: Early Transcendentals (C4). None of these references cover the material in exactly the same way we will cover topics in MAT186H1F. The main use of these textbooks is for homework problems. Note that C3 has no review of high school material, whereas C1 and C4 do. Thus in the first week of lectures, the main references will be C1 and C4. In weeks 3, 5, 6, 8, 9, 10, 11 and 12 the main reference will be C1. In week 13, the main reference will be C2. In weeks 2, 4 and 7, in which we "jump around" between differential and integral calculus, the best reference will probably be your lecture notes. In addition you should look over this **general advice** (GA) for first year math students.

Marking Scheme: Math Diagnostic Tests: 5%; WeBWorK Problem Sets: 5%; Assignments/Quizzes in Lectures: 5%; Tutorial Quizzes: 5%; Term Test 1: 20%; Term Test 2: 20%; Final Exam: 40%

Diagnostic Tests: to be written on Tuesday, Sep 10, between 1:10 and 3 PM. The purpose of these tests is to determine if any students have gaps in their high school math. Then we can offer help to these students before the *real* tests start. The Math Diagnostic Test is a 110 min multiple choice test covering six sections: Algebra, Functions, Graphing, Trigonometry, Exponentials and Logarithms, and Calculus Fundamentals. No calculators are permitted. Locations will be posted on Quercus. This test will count 5% of your final mark in MAT186H1F. Your mark is based on

the five parts, Algebra, Functions, Graphing, Trigonometry, Exponentials & Logarithms, with each part to count 1%. (The sixth part, Calculus Fundamentals will not count, it's just for our interest.) To get your 1% for each part you must get at least 8 of the 10 questions correct; if you get 7 or less, then your mark for that part is 0%. If you get 7 or less on any of the five parts, then you can rewrite those parts two weeks later, on Sep 24. There will also be some help sessions made available to students who want more practice on any of the five parts of the Diagnostic Tests before Sep 24.

Suggested Homework: the homework exercises listed in the weekly Lecture Schedule are practice problems that you should try on your own. They do not count, but some quiz, test or exam questions may well be similar to homework problems. You can get help with these problems in the Math Learning Centre.

Math Learning Centre PG101: this is a drop-in centre that you can visit to get help with your math problems. Any Teaching Assistant there should be able to help you, although it can get very busy. There will be specific MAT186H1F TA's available at certain times, to be posted. Here is a link to the Math Learning Centre website.

WeBWorK Problem Sets: WeBWorK is an on-line homework system. It can be accessed through the lecture website on Quercus. There will be five problem sets posted during the term that will each count 1%. Each problem set will be available for about 2 weeks. The due dates for the problem sets are: Sep 22, Oct 6, Oct 14, Nov 3 and Nov 25. There will also be an extra problem set posted at the end of the term, that won't count, but will cover some practice problems on new material prior to the exam. These problem sets will be the same for all students.

Assignments/Quizzes in Lectures: these depend totally on your lecturer, and may range from written assignments in essay form to on-line multiple choice questions in Quercus. You must participate in your lecture section, as found on your timetable, to get credit for this work.

Tutorial Quizzes: the schedule for the five tutorial quizzes is below, in the Tutorial Schedule. You must write the quiz in your own tutorial, as listed on your timetable, for it to count.

Term Test 1: a 100-minute term test is scheduled for Tuesday, Oct 15, between 1:15 and 2:55 PM in locations to be announced. This test will be the same for all students.

Term Test 2: a 100-minute term test is scheduled for Tuesday, Nov 26, between 1:15 and 2:55 PM in locations to be announced. This test will be the same for all students.

Final Exam: there will be a common final exam, 150 min long, during the exam period, Dec 6-20. This exam will be the same for all students.

Calculators: use of a Casio FX-991 or Sharp EL-520 calculator will be permitted during all quizzes, tests and exams, except for the Math Diagnostic Tests. However, it is still your responsibility to explain your work. A correct answer with no justification will receive no marks.

Course Coordinator: D. Burbulla. Office: GB 149

email: burbulla@math.toronto.edu; office hours: MR 10; TF 11; MWR 3; T 1-4[1]

[1]Not on Sep 10, Oct 1, Oct 15, Nov 12 or Nov 26

Lecture Schedule: the Engineering term consists of 13 weeks of classes spread over 14 calendar weeks. The first day of classes is Thursday, Sep 5 and the last day of classes is Wednesday, Dec 4. Each lecturer will cover the material in his or her own style. Not *everything* in the reference sections will necessarily be covered in lectures. Some weeks are almost entirely review of high school material; some weeks cover only new material; and some weeks combine review with new material. Instructors may become slightly ahead or behind schedule. In Week 6 seven lecture sections will miss a class due to the Thanksgiving holiday, so slightly less is scheduled for Week 6.

First Class, Sep 5 or 6: course orientation.

Week 1, Sep 9–13: Review: inverse functions; logarithmic, exponential and trigonometric functions. Sum and difference formulas for sine and cosine; double angle formulas; definition of the inverse trigonometric functions $\sin^{-1}, \cos^{-1}, \tan^{-1}, \sec^{-1}$, aka $\arcsin, \arccos, \arctan$, arcsec.

Ref: C1 Sec 1.3, 1.4, 1.5; C4 Sec 1.3, 2.3, 2.4, 2.5, 2.6.

Homework: C1 Sec 1.3 #153, 161, 171, 181; Sec 1.4 #193, 197, 201, 205, 217; Sec 1.5 #233, 235, 237, 243, 245, 279, 289, 291, 299, 307. C4 all exercises in Sec 1.3, 2.4, 2.5, 2.6

Week 2, Sep 16–20: Introduction to calculus: the tangent line problem and the area problem: definition of the tangent line to a curve as a limit of secant lines to the curve; average velocity and instantaneous velocity; the idea of a limit of a function; the idea of a derivative; definition of the area under a curve as the limit of a sum of rectangular approximations to the area under the curve; Riemann sums; the definite integral; velocity and displacement.

Ref: C1 Sec 2.1, 5.1, 5.2; C2 Sec 1.1, 1.2, 1.3, 4.1, 4.2; C3 Sec 2.1, 2.2, 5.3.

Homework: C1 Sec 2.1 #13, 14, 15, 20, 21; Sec 5.1 #14, 15, 16, 17, 23, 39, 43;
Sec 5.2 #61, 63, 65, 67, 69, 71, 73, 75, 81, 83.

Week 3, Sep 23–27: Types of limits and how to calculate them: informal definition of a limit; limit laws; basic algebraic techniques for calculating a limit in the 0/0 form—factoring and rationalizing; the squeeze law; the basic trig limit; one-sided limits; infinite limits and vertical asymptotes of a function; limits at infinity and horizontal asymptotes of a function.

Ref: C1 Sec 2.2, 2.3, 4.6 pp 407-425; C2 Sec 1.2, 1.3; C3 Sec 1.1, 1.3, 1.4, 1.6.

Homework: C1 Sec 2.2 #35, 36, 37, 46-67, 77, 79; Sec 2.3 #97, 99, 101, 103, 105, 115, 117; Sec 4.6 #263, 265, 267, 268, 269, 271, 273, 277, 281, 287. C3 Sec 1.3 #35, 36, 37, 38, 39, 40.

Week 4, Sep 30–Oct 4: Continuity: continuity of a function at a point; types of discontinuity—infinite, jump and removable; continuity on a closed interval; the Intermediate Value Theorem; isolating solutions to an equation by using IVT; definition of the derivative; differentiability of f at a implies continuity of f at a; definition of the Riemann Integral; basic properties of the Riemann Integral; continuity of f on $[a, b]$ implies existence of the definite integral of f on $[a, b]$.

Ref: C1 Sec 2.4, 3.1, 3.2, 5.1, 5.2; C2 Sec 1.3, 1.7, 4.1, 4.2, 4.3; C3 Sec 1.5, 5.2.

Homework: C1 Sec 2.4 #133, 135, 139, 141, 143, 145, 147, 149, 151, 153, 157, 161, 163, 165, 167; Sec 3.2 #61, 63, 65, 69, 71, 75, 77, 79. C3 Sec 1.5 #11-38; Sec 5.2 #19-26.

Week 5, Oct 7–11: Differentiation, mostly review of high school: the derivative function; slopes of tangent lines and normal lines to a curve; e and the derivative of exponential functions; rules of differentiation—derivatives of a sum, difference, product and quotient; higher order derivatives; derivatives of the trigonometric functions; derivatives as rates of change; position, velocity, acceleration; the chain rule; implicit differentiation; related rates problems.

Ref: C1 Sec 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.8, 4.1; C2 Sec 2.1, 2.2, 2.3, 2.4, 2.5, 2.7, 3.5; C3 Sec 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 4.2.

Homework: C1 Sec 3.3 #125, 127, 129, 141, 143, 147; Sec 3.4 #151, 157, 159, 165; Sec 3.5 #175, 181, 191, 193, 201, 209, 213; Sec 3.6 #223, 231, 233, 235, 239, 243, 249, 251, 257; Sec 3.8 #303, 307, 311, 313, 317, 321; Sec 4.1 #17, 19, 29, 31, 37, 39, 41.

C3 Sec 2.3 #33, 37; Sec 2.4 #19, 20, 29, 39, 44; Sec 2.5 #30, 32, 35, 39; Sec 2.6 #20, 29, 32, 38, 39.

Week 6, Oct 15-18: More about Differentiation: derivatives of logarithmic and exponential functions; logarithmic differentiation; derivatives of the inverse trigonometric functions; derivatives of inverse functions in general.

Ref: C1 Sec 3.7, 3.9; C2 Sec 2.6; C3 Sec 2.7.

Homework: C1 Sec 3.7 #269, 271, 273, 275, 277, 279, 281, 283, 285, 295, 297, 299; Sec 3.9 #333, 339, 341, 347, 349, 353, 355, 357. C3 Sec 2.7 #11, 12, 22, 23, 28.

Week 7, Oct 21–25: Anti-differentiation: antiderivatives; the indefinite integral; basic indefinite integration formulas; position, velocity, acceleration revisited; introduction to differential equations and initial value problems; exponential growth and decay; the Fundamental Theorem of Calculus—evaluating and differentiating definite integrals.

Ref: C1 Sec 4.10, 5.3 pp 552-559, 6.8; C2 Sec 4.3, 4.4, 5.2; C3 Sec 5.1, 5.2, 5.3, 5.4.

Homework: C3 Sec 5.1 #29-39; Sec 5.2 #7, 9, 11, 13; Sec 5.4 #41-50.

C1 Sec 4.10 #467, 481, 483, 493, 495, 501, 503, 505, 507, 509, 511; Sec 5.3 #149, 151, 153, 155, 157, 159, 161, 171, 173, 177, 185, 191, 193, 195, 197, 201; Sec 6.8 #353, 355, 369.

Week 8, Oct 28–Nov 1: Applications of the Derivative: maximum and minimum values; the Extreme Value Theorem; critical points; Rolle's Theorem; the Mean Value Theorem and its consequences; increasing and decreasing functions; the first derivative test; concavity, inflection points, and the second derivate test; curve sketching; optimization problems.

Ref: C1 Sec 4.3, 4.4, 4.5, 4.6, 4.7; C2 Sec 3.1, 3.2, 3.3, 3.4; C3 Sec 3.1, 3.2, 3.3, 3.4, 3.5, 4.3.

Homework: C3 Sec 3.1 #7, 8, 17-26.

C1 Sec 4.3 #91, 93, 95, 97, 101, 103, 119, 123, 125, 127, 131, 133, 141, 145; Sec 4.4 #149, 151, 165, 167, 169, 175, 177; Sec 4.5 #195, 199, 205, 207, 215, 217, 221, 227, 233, 241, 243, 245, 247, 249; Sec 4.6 #297, 301, 303, 305, 309; Sec 4.7 #315, 323, 327, 328, 341, 343, 345, 351.

Week 9, Nov 4–8: More Applications of the Derivative: linear approximations; Newton's Method for approximating solutions to the equation $f(x) = 0$; L'Hopital's Rule and indeterminate forms.

Ref: C1 Sec 4.2, 4.8, 4.9; C2 Sec 2.8; C3 Sec 4.1, 4.4.

Homework: C1 Sec 4.2 #49, 51, 55, 57, 59, 65, 69, 71, 73, 77, 79, 81, 83; Sec 4.8 #357, 363, 365, 369, 371, 375, 377, 379, 383, 385, 389, 395, 401, 405; Sec 4.9 #427, 431. C3 Sec 4.1 #9, 10, 12, 14.

Week 10, Nov 11–15: Integration: integrating even and odd functions; average value of a function; Mean Value Theorem for Integrals; change of variables in indefinite integrals; change of variables in definite integrals; integration by parts, time permitting.

Ref: C1 5.2 pp 541-543, 5.3 pp 549-552, 5.4 pp 572-574, 5.5, 5.6, 5.7; C3 Sec 5.4, 5.5, 5.6.

Homework: C3 Sec 5.6 #11, 17, 21, 37, 47. (These are all integration by parts problems.)

C1 Sec 5.2 #111, 113, 127, 129, 139; Sec 5.3 #199; Sec 5.4 #213, 219, 221, 231; Sec 5.5 #263, 265, 271, 275, 279, 281, 283, 295, 297, 307, 311, 317; Sec 5.6 #331, 333, 337, 349, 355, 357, 365; Sec 5.7 #391, 395, 399, 403, 411, 413, 415, 429, 431, 433.

Week 11, Nov 18–22: Applications of the Integral: velocity and displacement; average velocity and average speed; areas of regions between curves—integrating with respect to x or y; volume by slicing; solids of revolution; volumes by disc and washer methods.

Ref: C1 Sec 5.4 pp 566-572, 6.1, 6.2; C2 Sec 6.1, 6.2; C3 Sec 5.2, 6.1, 6.2.

Homework: C1 Sec 5.4 #223, 225; Sec 6.1 #3, 5, 7, 11, 15, 19, 21, 25, 27, 37, 41, 51, 53, 57; Sec 6.2 #67, 69, 71, 75, 81, 83, 89, 93, 99, 101, 103. C3 Sec 5.2 #15, 16, 17, 18.

Week 12, Nov 25–29: More Applications of the Integral: volumes by shells; arc length; surface area of a solid of revolution; density and mass; work; other (optional) applications .

Ref: C1 Sec 6.3, 6.4, 6.5, 6.6; C2 Sec 6.1, 6.2, 6.3, 6.4; C3 Sec 6.3, 6.4, 6.5.

Homework: C1 Sec 6.3 #125, 129, 133, 143, 145, 163; Sec 6.4 #173, 177, 193, 195, 199, 203, 207, 215; Sec 6.5 #223, 235, 239, 245, 251, 252, 253; Sec 6.6 #257, 259, 261, 265, 271, 273, 277. C3 Sec 6.5 #16, 24, 26.

Week 13, Dec 2–4: Yet More Applications of the Integral: functions defined in terms of definite integrals; the natural logarithm function defined as a definite integral; review

Ref: C2 Sec 5.1; C1 Sec 6.7.

Homework: C2 Sec 5.1 #3-7. C1 Sec 5.3 #163.

A Word About the Homework: the homework problems listed above are for your practice, and there are probably too many of them. You do not have to do all of them. If you are sure you know how to do a problem, skip it. But if you are not sure how to do a problem, then try it. The WeBWorK problems, which *do* count, will generally be more challenging than the homework problems listed above.

Tutorial Schedule: each tutorial will meet 12 times during the term. Tutorials begin on Monday, Sep 9 and end on Monday, Dec 2. The University is closed on Thanksgiving Monday, Oct 14. So the first five tutorial cycles run from Monday, Sep 9 to Friday, Oct 11; the last seven tutorial cycles run from Tuesday, Oct 15 until Monday, Dec 2. Tutorials are roughly one week behind the lecture schedule. The actual content in each tutorial, and what a quiz covers, will be posted on the course webpage before each cycle of tutorials. To get marks for tutorial quizzes you must attend your own tutorial.

1. **Sep 9–13:**

   Ref: GA, good and bad notation.

2. **Sep 16–20:** Quiz 1

   Ref: C1 Sec 1.3, 1.4; C4 Sec 2.6

3. **Sep 23–27:**

   Ref: C1 Sec 2.1, 5.1, 5.2; C2 Sec 1.1, 1.3, 4.1, 4.2; C3 Sec 2.1, 2.2, 5.3

4. **Sep 30–Oct 4:** Quiz 2

   Ref: C1 Sec 2.2, 2.3, 4.6 pp 407-425; C3 Sec 1.1, 1.3, 1.4, 1.6.

5. **Oct 7–11:**

   Ref: C1 Sec 2.4, 3.1, 3.2, 5.1, 5.2; C3 Sec 1.5, 5.2.

6. **Oct 15–21:**

   Ref: C1 Sec 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.8, 4.1; C3 Sec 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 4.2.

7. **Oct 22–28:** Quiz 3

   Ref: C1 Sec 3.7, 3.9; C3 Sec 2.7.

8. **Oct 29–Nov 4:**

   Ref: C1 Sec 4.10, 5.3 pp 552-559, 6.8; C3 Sec 5.1, 5.2, 5.4.

9. **Nov 5–11:**

   Ref: C1 Sec 4.3, 4.4, 4.5, 4.6, 4.7; C3 Sec 3.1, 3.2, 3.3, 3.4, 3.5, 4.3.

10. **Nov 12–18:** Quiz 4

    Ref: C1 Sec 4.2, 4.8, 4.9; C3 Sec 4.1, 4.4;

11. **Nov 19–25:** Quiz 5

    Ref: C1 Sec 5.2 pp 541-543, 5.3 pp 549-552, 5.4 pp 572-574, 5.5, 5.6, 5.7; C3 Sec 5.4, 5.5, 5.6.

12. **Nov 26–Dec 2:**

    Ref: C1 Sec 5.4 pp 566-572, 6.1, 6.2; C3 Sec 5.2, 6.1, 6.2.

Learning Outcomes (short version): students should be able to do the following:

1. Define limit, derivative and definite integral; and be able to calculate each of them.
2. Write well-written, well-explained solutions to given problems using correct notation.
3. Given the graph of one of f, f' or f", figure out the graphs of any one of the others.
4. Apply methods of calculus to solve optimization, related rates and physics problems.
5. Apply methods of calculus to calculate lengths, areas, volumes and average values.
6. Apply concepts and methods of calculus to analyze a real-world situation without cues.

Learning Outcomes (long version):

- Given a problem, be it as simple as a calculation or as complicated as a long, involved word problem, students should be able to write a well-organized solution that defines any variables used, describes any assumptions made, includes diagrams that illustrate the connection between variables, uses correct mathematical notation, and provides full explanation of all the steps involved. In particular, students should be able to give complete solutions to the following types of word problems:
    1. related rates problems
    2. optimization problems that can be reduced to one independent variable
    3. physics problems that involve position, velocity, acceleration, density, mass or work
    4. initial value problems with one differential equation in terms of one independent parameter
    5. exponential growth and decay problems
- Given a function $y = f(x)$, students should be able to
    1. determine if f is one-to-one, and if it is one-to-one, to calculate the inverse of f
    2. determine if f is even or odd, or neither
    3. find and classify all the discontinuities of f
    4. define the derivative of f at $x = a$
    5. define and calculate the equations of the tangent line and the normal line to f at $x = a$
    6. use the tangent line of f at $x = a$ to obtain a linear approximation of f near a
    7. solve geometric problems that involve tangent lines and normal lines of f
    8. calculate the first and second derivative of f
    9. find all the asymptotes—vertical, slant or horizontal—to the graph of f
    10. find all the critical points and inflection points of f and determine if the critical points are maximum points or minimum points of f
    11. find the intervals on which f is increasing, decreasing, concave up or concave down
    12. sketch the graph of f, labelling all the asymptotes, maximum or minimum points, inflection points, and vertical tangents of f
    13. approximate roots of the equation $f(x) = 0$, using Newton's method
    14. define an antiderivative of f on the interval $[a, b]$

15. define the indefinite integral of f
16. find the indefinite integral of f by using appropriate algebraic manipulations or by making a change of variable, or by using integration by parts, or a combination of all of these
17. define the definite integral of f on the interval $[a, b]$
18. approximate the definite integral of f on the interval $[a, b]$ by using Riemann sums
19. calculate the definite integral of f on the interval $[a, b]$ by using the Fundamental Theorem of Calculus
20. solve problems and prove statements about definite integrals using properties of definite integrals
21. calculate the average value of f on the interval $[a, b]$
22. calculate the length of the curve $y = f(x)$ on the interval $[a, b]$
23. calculate the surface area of the solid of revolution obtained by revolving $y = f(x)$, on the interval $[a, b]$, about the x-axis, or about the y-axis
24. use the method of discs to calculate the volume of the solid of revolution obtained by revolving $y = f(x)$, on the interval $[a, b]$, about an axis parallel to the x-axis
25. use the method of shells to calculate the volume of the solid of revolution obtained by revolving $y = f(x)$, on the interval $[a, b]$, about an axis parallel to the y-axis

- Given a limit in one of the following indeterminate forms, $0/0, \infty/\infty, \infty - \infty, 0 \cdot \infty, 1^\infty, 0^0$ or ∞^0, students should be able to calculate the limit by factoring, rationalizing, or other algebraic manipulation; or by applying L'Hopital's Rule or the Squeeze Law, as appropriate

- Given the graph of any one of f, f' or f'', students should be able to

  1. figure out possible graphs of the other two
  2. decide whether or not a given graph is a possible graph for either of the other two

- Given the velocity function in terms of time t of a particle moving along a straight line, students should be able to set up and evaluate definite integrals to find

  1. the net change of position of the particle on the time interval $[a, b]$
  2. the total distance travelled by the particle on the time interval $[a, b]$
  3. the average velocity of the particle on the time interval $[a, b]$
  4. the average speed of the particle on the time interval $[a, b]$

- Given a region in the xy-plane, bounded by one or more curves, students should be able to

  1. find the area of the region by setting up and evaluating one or more appropriate definite integrals with respect to x, or with respect to y, whichever is more suitable
  2. find the volume of the solid of revolution obtained by revolving the region about a given axis parallel to the x- or y-axis, by setting up and evaluating one or more appropriate definite integrals with respect to x, or with respect to y, whichever is more suitable