MAT188H1F - Linear Algebra - Fall 2019

Solutions to Term Test 2 - November 12, 2019

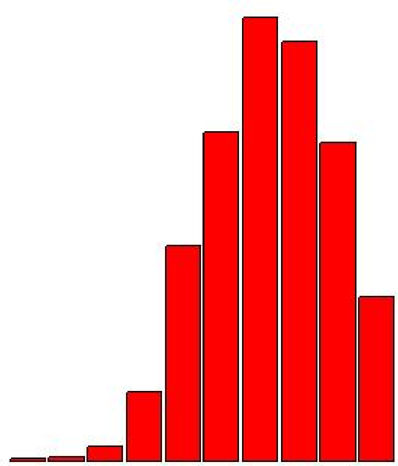
Time allotted: 100 minutes. Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

Genreal Comments:

- The range on every question was 0 to perfect.
- There were two bonus marks available, one each in Question 8(b) and 8(d): the bonus mark was for finding a counterexample.
- Many students missed the connection between parts (a) and (b) of Question 2, which meant some students did way too much work for that question.
- In Question 7 many students used 2×2 matrices in their 'proofs.' This is *not* valid: you can't prove a general statement by taking a special case. Otherwise I could prove all prime numbers are even: $n = 2$ is prime and it is even.

Breakdown of Results: 920 registered students wrote this test. The marks ranged from 6.25% to 102.5%—because of the two bonus marks—and the average was 66.9%. Some statistics on grade distribution are in the table on the left, and a histogram of the marks (by decade) is on the right.

Grade	%	Decade	%
		90-100%	8.3%
A	24.3%	80-89%	16.1%
B	21.2%	70-79%	21.2%
C	22.4%	60-69%	22.4%
D	16.6%	50-59%	16.6%
F	15.5%	40-49%	10.9%
		30-39%	3.5%
		20-29%	0.8%
		10-19%	0.2%
		0-9%	0.1%



1. [10 marks; avg: 9.5/10] Let

$$A = \begin{bmatrix} 1 & 0 \\ 3 & -2 \\ 0 & 3 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 2 \end{bmatrix}.$$

Compute the following:

(a) [3 marks] $A\,B$

Solution:

$$AB = \begin{bmatrix} 1 & 0 \\ 3 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 5 & -3 & 2 \\ -3 & 0 & 6 \end{bmatrix}$$

(b) [3 marks] $B\,A$

Solution:

$$BA = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ -1 & 6 \end{bmatrix}$$

(c) [4 marks] $A^T\,B^T$

Solution: quick way is

$$A^T B^T = (BA)^T = \begin{bmatrix} -2 & 8 \\ -1 & 6 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 \\ 8 & 6 \end{bmatrix}.$$

Long way is

$$A^T B^T = \begin{bmatrix} 1 & 0 \\ 3 & -2 \\ 0 & 3 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 8 & 6 \end{bmatrix}.$$

2. [avg: 6.9/10] Let $A = \begin{bmatrix} 3 & c & 5 \\ 0 & 2 & c \\ -1 & 0 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

(a) [4 marks] Find all values of c such that $\det(A) = 0$.

Solution: $\det \begin{bmatrix} 3 & c & 5 \\ 0 & 2 & c \\ -1 & 0 & 1 \end{bmatrix} = 6 - c^2 + 10 = 16 - c^2$. So $\det(A) = 0 \Leftrightarrow c = \pm 4$.

(b) [6 marks] Find all values of c for which the system $A\vec{x} = \vec{b}$ has

(i) no solution, (ii) a unique solution, (iii) infinitely many solutions.

Answers: (i) $c = -4$ (ii) $c \neq \pm 4$ (iii) $c = 4$

Solution: if $c \neq \pm 4$, then A is invertible and the system $A\vec{x} = \vec{b}$ has a unique solution.

If $c = 4$, reducing the augmented matrix of the system gives

$$\left[\begin{array}{ccc|c} 3 & 4 & 5 & -1 \\ 0 & 2 & 4 & 1 \\ -1 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 4 & 8 & 2 \\ 0 & 2 & 4 & 1 \\ -1 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 1 \\ -1 & 0 & 1 & 1 \end{array}\right].$$

There is one free variable, so in this case there will be infinitely many solutions.

If $c = -4$, reducing the augmented matrix of the system gives

$$\left[\begin{array}{ccc|c} 3 & -4 & 5 & -1 \\ 0 & 2 & -4 & 1 \\ -1 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & -4 & 8 & 2 \\ 0 & 2 & -4 & 1 \\ -1 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 0 & 0 & 4 \\ 0 & 2 & -4 & 1 \\ -1 & 0 & 1 & 1 \end{array}\right].$$

The top row indicates that in this case the system has no solution.

Alternate approach: row reduce the augmented matrix.

$$\left[\begin{array}{ccc|c} 3 & c & 5 & -1 \\ 0 & 2 & c & 1 \\ -1 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & 0 & 1 & 1 \\ 0 & 2 & c & 1 \\ 3 & c & 5 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & 0 & 1 & 1 \\ 0 & 2 & c & 1 \\ 0 & c & 8 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & 0 & 1 & 1 \\ 0 & 2 & c & 1 \\ 0 & 0 & 16 - c^2 & 4 - c \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} -1 & 0 & 1 & 1 \\ 0 & 2 & c & 1 \\ 0 & 0 & (4 - c)(4 + c) & 4 - c \end{array}\right]$$

and then analyze as before.

3. [avg: 9.4/10] Consider the system of equations (*) $\begin{cases} x_1 + 2x_2 + x_3 = 7 \\ x_1 + 3x_2 + x_3 = 7 \\ 2x_1 + 7x_2 + 3x_3 = 19 \end{cases}$.

(a) [3 marks] Write this system as a single matrix equation $A\vec{x} = \vec{b}$. Clearly identify $A, \vec{x}$ and $\vec{b}$.

Solution:

$$\underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 2 & 7 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 7 \\ 7 \\ 19 \end{bmatrix}}_{\vec{b}}$$

(b) [4 marks] Find A^{-1}.

Solution: use the Gaussian algorithm to find A^{-1} :

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 2 & 7 & 3 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 3 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right] = [I|A^{-1}]$$

That is,

$$A^{-1} = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix}.$$

(c) [3 marks] Use A^{-1} to solve the system of equations (*).

Solution: $A\vec{x} = \vec{b} \Rightarrow \vec{x} = A^{-1}\vec{b}$, so

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 7 \\ 19 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

4. [10 marks; avg: 7.5/10] Solve the following system of linear differential equations

$$\begin{array}{rcrcr} f_1' &=& 2f_1 &+& 5f_2 \\ f_2' &=& f_1 &-& 2f_2 \end{array}$$

for f_1 and f_2 as functions of x if $f_1(0) = 2$ and $f_2(0) = 10$.

Solution: the coefficient matrix is $A = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$, for which

$$\det(\lambda I - A) = (\lambda - 2)(\lambda + 2) - 5 = \lambda^2 - 9 = (\lambda - 3)(\lambda + 3).$$

The eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = -3$, the roots of the characteristic polynomial. To find the eigenvectors, find a (simple) non-zero solution $\vec{v}$ to the homogeneous system $(\lambda I - A)\vec{v} = \vec{0}$:

For eigenvalue $\lambda_1 = 3$:

$$\left[\begin{array}{rr|r} 1 & -5 & 0 \\ -1 & 5 & 0 \end{array}\right] \to \left[\begin{array}{rr|r} 1 & -5 & 0 \\ 0 & 0 & 0 \end{array}\right];$$

take

$$\vec{v}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}.$$

For eigenvalue $\lambda_2 = -3$:

$$\left[\begin{array}{rr|r} -5 & -5 & 0 \\ -1 & -1 & 0 \end{array}\right] \to \left[\begin{array}{rr|r} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right];$$

take

$$\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

So the general solution is

$$\begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{3x} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-3x}.$$

Use the initial conditions at $x = 0$ to find c_1, c_2 :

$$\begin{bmatrix} 2 \\ 10 \end{bmatrix} = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 2 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 10 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ 48 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}.$$

Thus

$$f_1(x) = 10\, e^{3x} - 8\, e^{-3x} \text{ and } f_2(x) = 2\, e^{3x} + 8\, e^{-3x}.$$

5. [avg: 4.8/10]

5.(a) [5 marks] Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear transformation defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} z \\ y \\ x \end{bmatrix}$. Find the matrix of T and show it is diagonalizable.

Solution: let the matrix of T be $A = [T(\vec{e}_1)\ T(\vec{e}_2)\ T(\vec{e}_3)]$. Then

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } \det(\lambda I - A) = \det \begin{bmatrix} \lambda & 0 & -1 \\ 0 & \lambda - 1 & 0 \\ -1 & 0 & \lambda \end{bmatrix} = (\lambda - 1)(\lambda^2 - 1) = (\lambda + 1)(\lambda - 1)^2.$$

For the repeated eigenvalue $\lambda = 1$ there are two basic eigenvectors:

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]; \text{ take } \vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Thus A is diagonalizable. (You don't actually have to diagonalize A, but of course you could. That would be another way to show that it is diagonalizable.)

5.(b) [5 marks] Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation which is a reflection in the line with equation $y = -x$, followed by a reflection in the x-axis. Show that T is a rotation. What is the angle of rotation?

Solution: if you recall the formulas for the matrices, then

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\text{reflection in x- axis}} \quad \text{times} \quad \underbrace{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}_{\text{reflection in line y=-x}} = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{rotation of pi/2}}.$$

OR, just follow effect on $\vec{e}_1$ and $\vec{e}_2$:

$$\vec{e}_1 \underbrace{\longrightarrow}_{\text{reflection in line y=-x}} -\vec{e}_2 \underbrace{\longrightarrow}_{\text{reflection in x- axis}} \vec{e}_2$$

and

$$\vec{e}_2 \underbrace{\longrightarrow}_{\text{reflection in line y=-x}} -\vec{e}_1 \underbrace{\longrightarrow}_{\text{reflection in x- axis}} -\vec{e}_1.$$

So the matrix of the composition is

$$[\vec{e}_2 \quad -\vec{e}_1] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

6. [10 marks; avg: 7.5/10] Suppose $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ -6 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -9 \end{bmatrix}.$$

(a) [6 marks] Find a 2×2 matrix A such that $T(\vec{x}) = A\,\vec{x}$, for all $\vec{x}$ in $\mathbb{R}^2$.

Solution: let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $T(\vec{x}) = A\,\vec{x}$. In particular,

$$A\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix} \text{ and } A\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ -9 \end{bmatrix}.$$

We can combine these two equations into one matrix equation and then solve for A :

$$A\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -6 & -9 \end{bmatrix} \Leftrightarrow A = \begin{bmatrix} -2 & -1 \\ -6 & -9 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & -1 \\ -6 & -9 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ 3 & -3 \end{bmatrix}.$$

Alternate Approach: observe that $\vec{e}_1 = 4\begin{bmatrix} 1 \\ 3 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\vec{e}_2 = -\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, so

$$T(\vec{e}_1) = 4T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) - 3T\left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = 4\begin{bmatrix} -2 \\ -6 \end{bmatrix} - 3\begin{bmatrix} -1 \\ -9 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

and

$$T(\vec{e}_2) = -T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) + T\left(\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = -\begin{bmatrix} -2 \\ -6 \end{bmatrix} + \begin{bmatrix} -1 \\ -9 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}.$$

(b) [4 marks] What is $T^{-1}\left(\begin{bmatrix} 24 \\ 36 \end{bmatrix}\right)$?

Solution: the matrix of T^{-1} is A^{-1}, so

$$T^{-1}\left(\begin{bmatrix} 24 \\ 36 \end{bmatrix}\right) = \begin{bmatrix} -5 & 1 \\ 3 & -3 \end{bmatrix}^{-1}\begin{bmatrix} 24 \\ 36 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} -3 & -1 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 24 \\ 36 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -9 \\ -21 \end{bmatrix}.$$

7. [avg: 2.7/10]

7.(a) [3 marks] Prove: if A is an $n \times n$ matrix then $A + A^T + A^T A$ is symmetric.

Solution: use properties of transpose.

$$(A + A^T + A^T A)^T = A^T + (A^T)^T + (A^T A)^T = A^T + A + A^T (A^T)^T = A + A^T + A^T A.$$

So $A + A^T + A^T A$ is its own transpose; it is symmetric.

7.(b) [4 marks] Prove: if $\vec{v}$ is an eigenvector of the $n \times n$ matrix A with eigenvalue λ then $\vec{v}$ is an eigenvector of A^3 with corresponding eigenvalue λ^3.

Solution: we know $A\,\vec{v} = \lambda\,\vec{v}$. So

$$A^2\,\vec{v} = A(A\,\vec{v}) = A(\lambda\,\vec{v}) = \lambda(A\,\vec{v}) = \lambda(\lambda\,\vec{v}) = \lambda^2\,\vec{v}$$

and similarly

$$A^3\,\vec{v} = A(A^2\,\vec{v}) = A(\lambda^2\,\vec{v}) = \lambda^2(A\,\vec{v}) = \lambda^2(\lambda\,\vec{v}) = \lambda^3\,\vec{v}.$$

Thus $\vec{v}$ is an eigenvector of A with corresponding eigenvalue λ^3.

7.(c) [3 marks] Prove: if A is an $n \times n$ matrix then A and A^T have the same characteristic polynomial (and so the same eigenvalues.)

Solution: use the fact that a square matrix and its transpose have the same determinant. Thus

$$\det(\lambda I - A^T) = \det(\lambda I^T - A^T) = \det\left((\lambda I - A)^T\right) = \det(\lambda I - A).$$

Conclude: the characteristic polynomial of A^T is the same as the characteristic polynomial of A.

8. [avg: 5.3/10] Indicate if the following statements are **True** or **False**, and give a *brief* explanation why.

(a) [2 marks] If $\lambda = 0$ is an eigenvalue of the $n \times n$ matrix A, then A is not invertible.

⊗ **True** ○ **False**

Explanation: if $\vec{v} \neq \vec{0}$ is a corresponding eigenvector, then $A\,\vec{v} = \vec{0}$. This means the homogeneous system has a non-trivial solution, which means A is not invertible.

OR

$0 = \det(A - 0I) = \det(A)$, so A is not invertible.

(b) [2 marks] If A, P and D are $n \times n$ matrices, D is a diagonal matrix, and $AP = PD$, then A is diagonalizable.

○ **True** ⊗ **False**

Explanation: it's true if P is invertible. But if P is not invertible, A need not be diagonalizable. For example, take

$$P = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\ D = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then A is not diagonalizable but $AP = PD = O$.

(c) [2 marks] If $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation then $T(\vec{0}) = \vec{0}$.

⊗ **True** ○ **False**

Explanation: let the matrix of T be A. Then $T(\vec{0}) = A\,\vec{0} = \vec{0}$.

(d) [2 marks] If A and B are $n \times n$ matrices such that A is invertible, then $ABA^{-1} = B$.

○ **True** ⊗ **False**

Explanation: the conclusion is equivalent to $AB = BA$. But this is not always true. For example if

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},\ B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix},$$

then A is invertible but

$$AB = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = BA.$$

(e) [2 marks] If A is an $n \times n$ matrix such that $A^5 - 3A - I = O$, where I is the $n \times n$ identity matrix and O is the $n \times n$ zero matrix, then A is invertible.

⊗ **True** ○ **False**

Explanation: $A^5 - 3A = I \Rightarrow A(A^4 - 3I) = I$, so $A^{-1} = A^4 - 3I$.

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