MAT187 - Calculus II - Winter 2015

Term Test 1 - February 3, 2015

Time allotted: 100 minutes. Aids permitted: None.

Total marks: 50

Full Name: ______________________________________
Last First

Student Number: ______________________________________

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Instructions

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- Please have your **student card** ready for inspection, turn off all cellular phones, and read all the instructions carefully.
- DO NOT start the test until instructed to do so.
- This test contains 14 pages (including this title page). Make sure you have all of them.
- You can use pages 12–14 for rough work or to complete a question (**Mark clearly**).
  DO NOT DETACH PAGES 12–14.

GOOD LUCK!

PART I No explanation is necessary. **(10 marks)**

1. Consider the solid of revolution generated by revolving the region between two functions $f(x) \leqslant g(x)$ for $x \in [a, b]$ around the $x-$axis. Then its volume is given by (circle **one** choice)

(a) $\displaystyle\int_a^b \big(g(x) - f(x)\big)\,dx$ **(c)** $\displaystyle\int_a^b \pi\big(g(x) - f(x)\big)^2\,dx$

(b) $\displaystyle\int_a^b 2\pi x\big(g(x) - f(x)\big)\,dx$ **(d)** $\displaystyle\int_a^b \pi\big(g(x)^2 - f(x)^2\big)\,dx$

2. Consider $\displaystyle\int_0^{\frac{\pi}{2}} \sin^{83} x \cos^{83} x\,dx$ and make a substitution to obtain

$$\int_0^{\frac{\pi}{2}} \sin^{83} x \cos^{83} x\,dx = \int_a^b f(u)\,du.$$

The substitution is

$u =$ ______________________________

$a =$ ______________________________

$b =$ ______________________________

3. On the integral of question **2**, the integrand becomes

$f(u) =$ ______________________________

4. A radioactive material decayed by 10% in 50 years.

Its half-life is ______________________________.

5. Let $a > 0$ and consider the region bounded by the graph of $y = ae^{-ax}$ and the $x-$axis on the interval $[0, \infty)$.

Its area is ______________________________.

6. Consider the rational function $\dfrac{4x^2-2x^2+x}{(x+1)(x-2)^3(x^2+9)^2}$. When using **partial fractions**, we write this function as a sum of the following terms (**circle all that apply**):

(a) $\dfrac{A}{x}$	**(d)** $\dfrac{D}{(x+1)}$	**(g)** $\dfrac{G}{(x-2)}$	**(j)** $\dfrac{J}{(x^2+9)}$	**(m)** $\dfrac{Mx+N}{(x^2+9)}$
(b) $\dfrac{B}{x^2}$	**(e)** $\dfrac{E}{(x+1)^2}$	**(h)** $\dfrac{H}{(x-2)^2}$	**(k)** $\dfrac{K}{(x^2+9)^2}$	**(n)** $\dfrac{Ox+P}{(x^2+9)^2}$
(c) $\dfrac{C}{x^3}$	**(f)** $\dfrac{F}{(x+1)^3}$	**(i)** $\dfrac{I}{(x-2)^3}$	**(l)** $\dfrac{L}{(x^2+9)^3}$	**(o)** $\dfrac{Qx+R}{(x^2+9)^3}$

7. Consider two functions $f(x)$ and $g(x)$ satisfying $0 \leqslant f(x) \leqslant g(x)$ for $x \in (0, \infty)$.
Assume that $\displaystyle\int_1^\infty g(x)\,dx$ **converges**. Then $\displaystyle\int_1^\infty f(x)\,dx$

(a) converges **(b)** diverges **(c)** we cannot tell

8. Consider two functions $f(x)$ and $g(x)$ satisfying $0 \leqslant f(x) \leqslant g(x)$ for $x \in (0, \infty)$.
Assume that $\displaystyle\int_1^\infty g(x)\,dx$ **diverges**. Then $\displaystyle\int_1^\infty f(x)\,dx$

(a) converges **(b)** diverges **(c)** we cannot tell

9. Recall that when approximating the integral $\displaystyle\int_a^b f(x)\,dx$ using the trapezoid rule, we make an error of at most $E_T \leqslant \frac{K(b-a)}{12}(\Delta x)^2$, where $K = \max\limits_{x\in[a,b]} |f''(x)|$ and $\Delta x = \frac{b-a}{n}$.

To approximate the integral $\displaystyle\int_0^1 e^{(x^2)}\,dx$ with a maximum error of $\frac{e}{32}$, I should choose

$n \geqslant$ ______________________________.

10. A free-hanging rope forms a catenary: a curve which satisfies

$$y''(x) = \frac{1}{a}\sqrt{1+\big(y'(x)\big)^2} \qquad \text{for } x \in [-b, b].$$

Assume that for this rope, $y'(b) = -y'(-b) = \frac{10}{a}$. Then the length of the rope is

$$L = \int_{-b}^{b} \sqrt{1+\big(y'(x)\big)^2}\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

(express the length as a number explicitly)

PART II **Justify** your answers.

11. You are working at a biology lab with a population of bacteria which grows **(10 marks)**

proportionally to its population. Moreover, the population doubles its size every hour.

(a) Assuming that you start with P_0 million bacteria, find a formula for the population of bacteria after t hours.

(b) You start with 100 million bacteria and you have two containers. Each can hold 300 million bacteria. Your job is to grow as many bacteria as you can in 2 hours.

What is the best way to divide the bacteria in the two containers? Justify your answer.

(**Hint.** This question is not hard)

12. Compute the following integrals. **(10 marks)**

(a) Let $b, \omega > 0$. Calculate $\displaystyle\int_0^b e^{-x} \sin(\omega x)\, dx$.

(b) Calculate $\displaystyle\int_0^1 \frac{\arcsin(x)\sqrt{1-x^2}}{\cos\big(\arcsin(x)\big)}\,dx.$

(**Hint.** Use a substitution)

13. Let $u(t)$ be the temperature in ^oC at the Pearson airport t years after March 1, 2000. **(10 marks)**

Then the average temperature for the first decade (2000-2010) is

$$\text{Average temperature} = \frac{1}{10}\int_0^{10} u(t)\,dt.$$

(a) Let $a < b$. What is the average temperature from <u>March 1</u> of the year $2000 + a$ to <u>September 1</u> of the year $2000 + b$?

(b) Assume that $u(t) = 5 + 30e^{-t}\sin(2\pi t)$. If this temperature pattern holds forever, what is the limiting average temperature?

14. Consider the function $f(x) = \dfrac{p}{x^p}$. Consider the solid created by rotating this function around the $x-$axis over the interval $[1, \infty)$.

(a) Calculate the volume of the solid. **(7 marks)**

(b) Find the value of p that minimizes the volume of this solid. **(3 marks)**

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