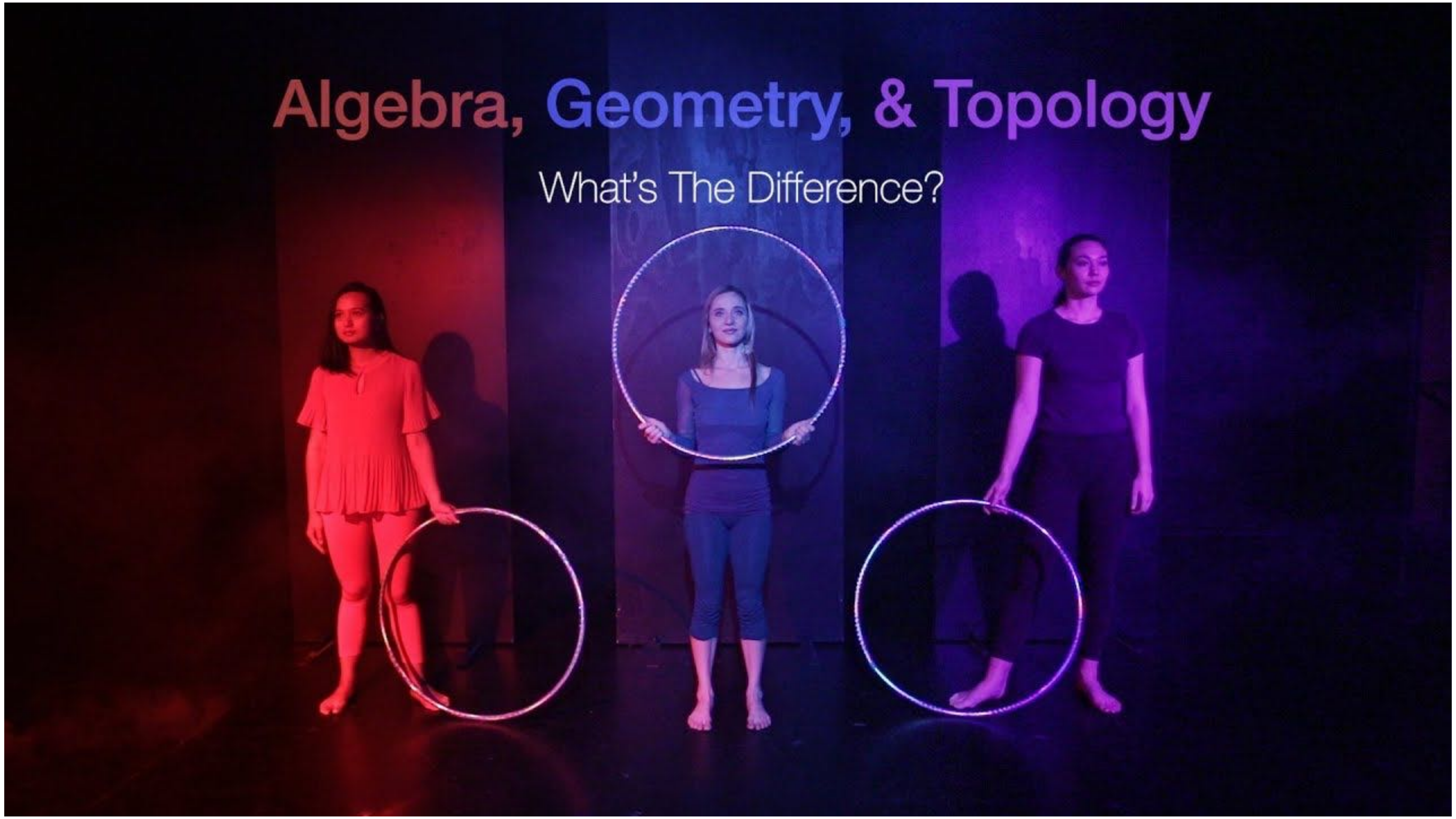
Is there math after calculus?

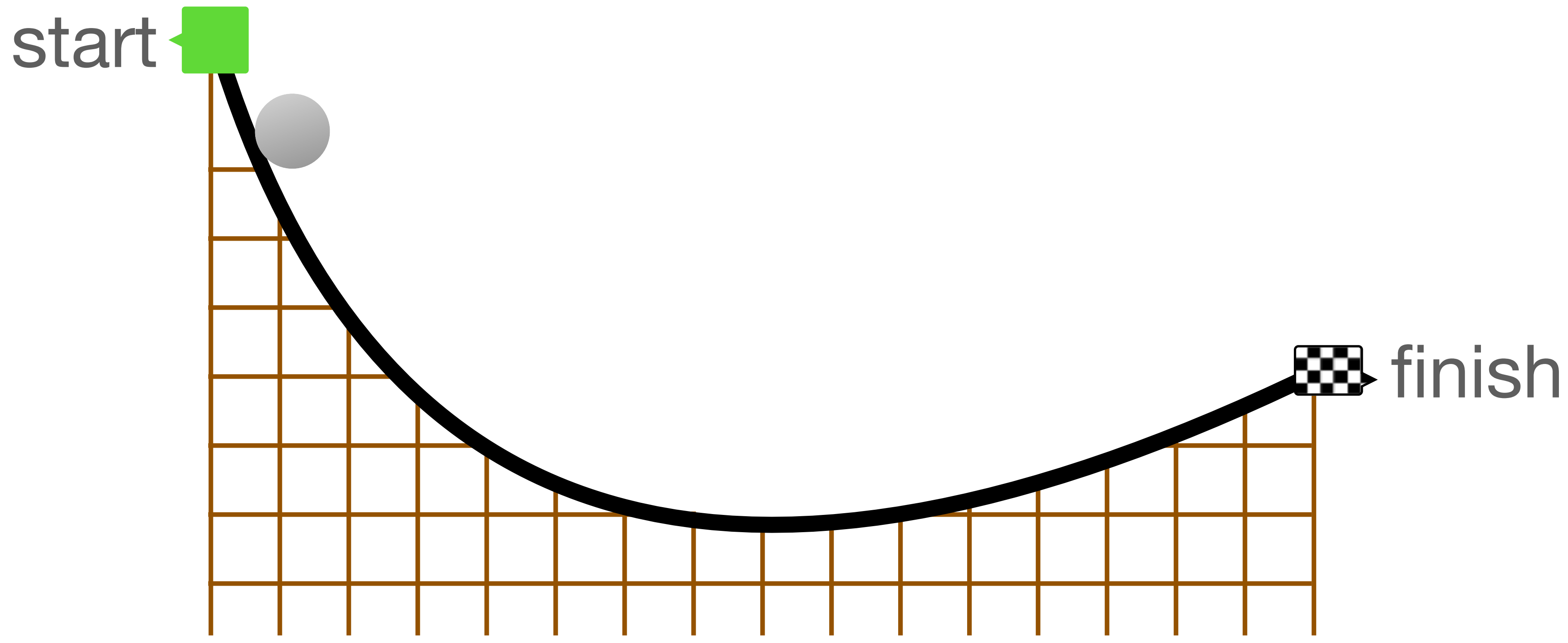
Winter 2023

MAT136 Calculus 2



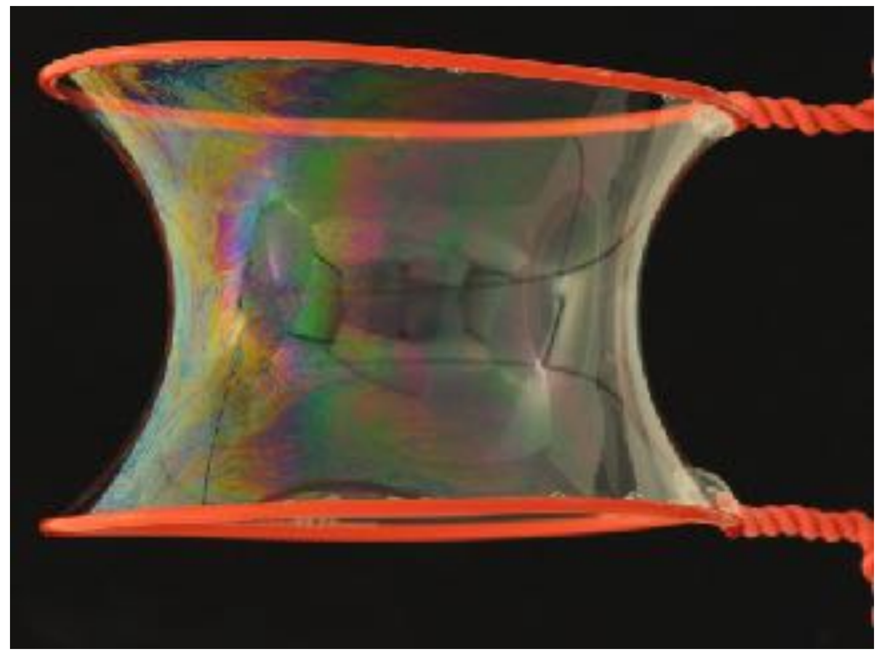
https://www.youtube.com/watch?v=xgKc7dFz-ko

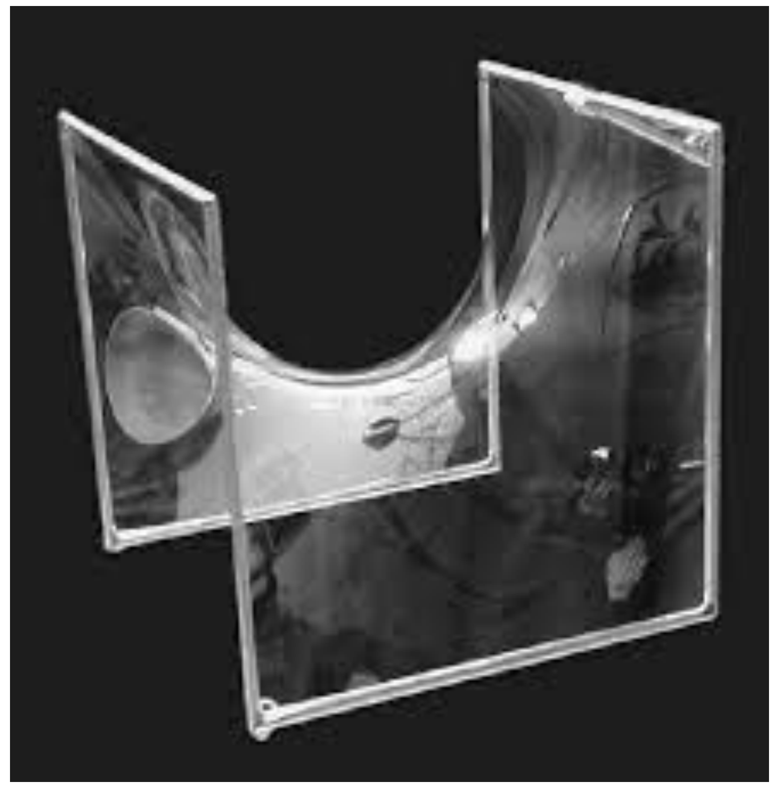
Calculus of variations



Brachistochrone Problem

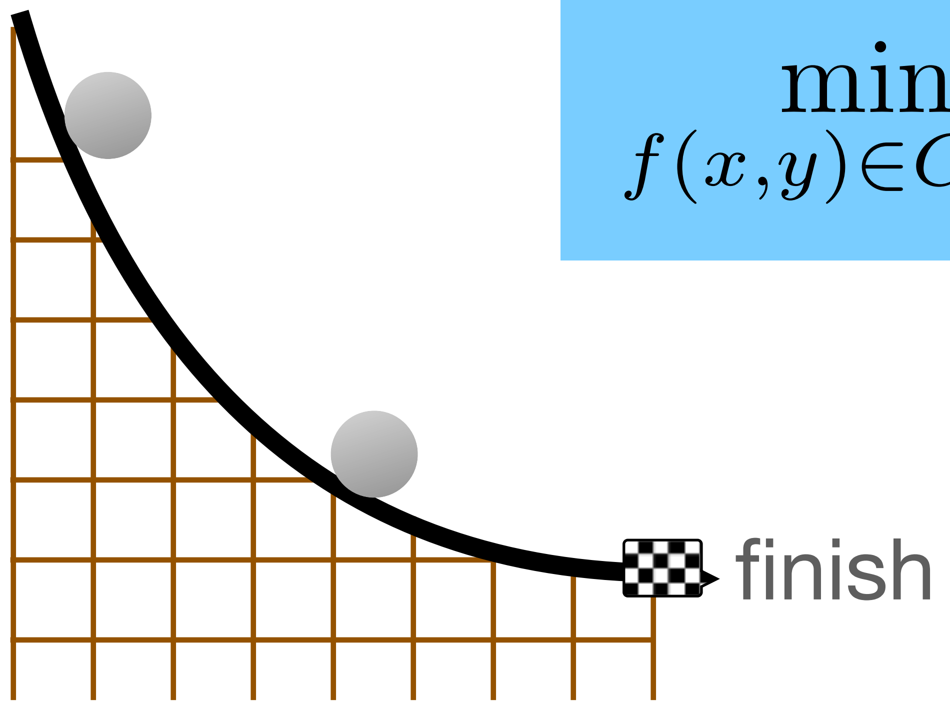
$$\min_{f \in C([a,b])} \int_a^b \sqrt{\frac{1 + \left(f'(x)\right)^2}{2gf(x)}}\, dx$$





Minimal Surfaces

$$\min_{f(x,y) \in C(D)} \left[\iint_D \sqrt{1 + \nabla f(x,y) \cdot \nabla f(x,y)}\, dx\, dy \right]$$



Tautochrone Problem

$$\frac{d}{da} \left[\int_a^b \sqrt{\frac{1 + \left(f'(x)\right)^2}{2gf(x)}}\, dx \right] = 0$$

classical theory for phase transitions for non-interacting fluid

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Ω container

classical theory for phase transitions for non-interacting fluid

Ω container

u fluid density satisfies

mass: $$\int_\Omega u\,dx = V$$

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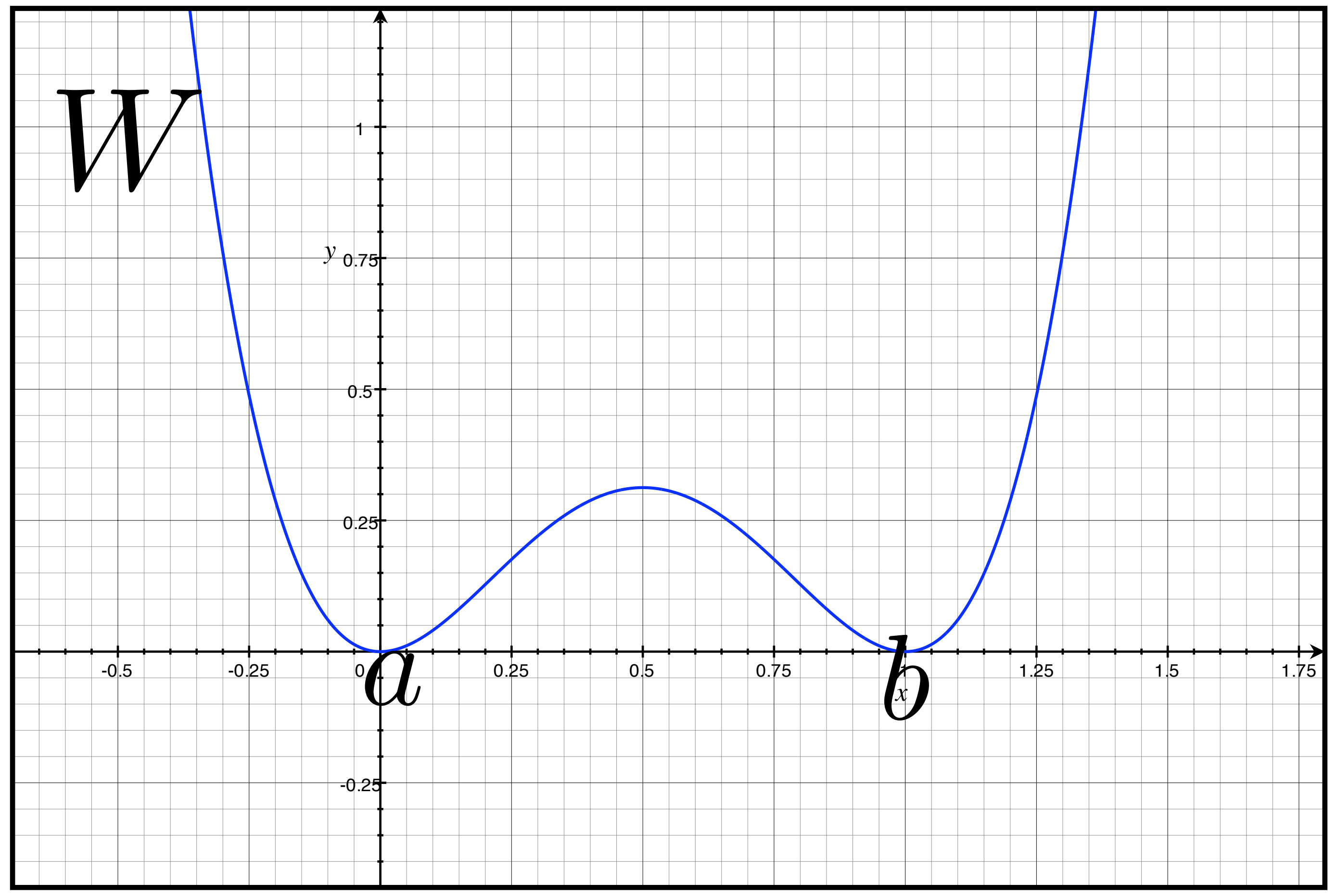
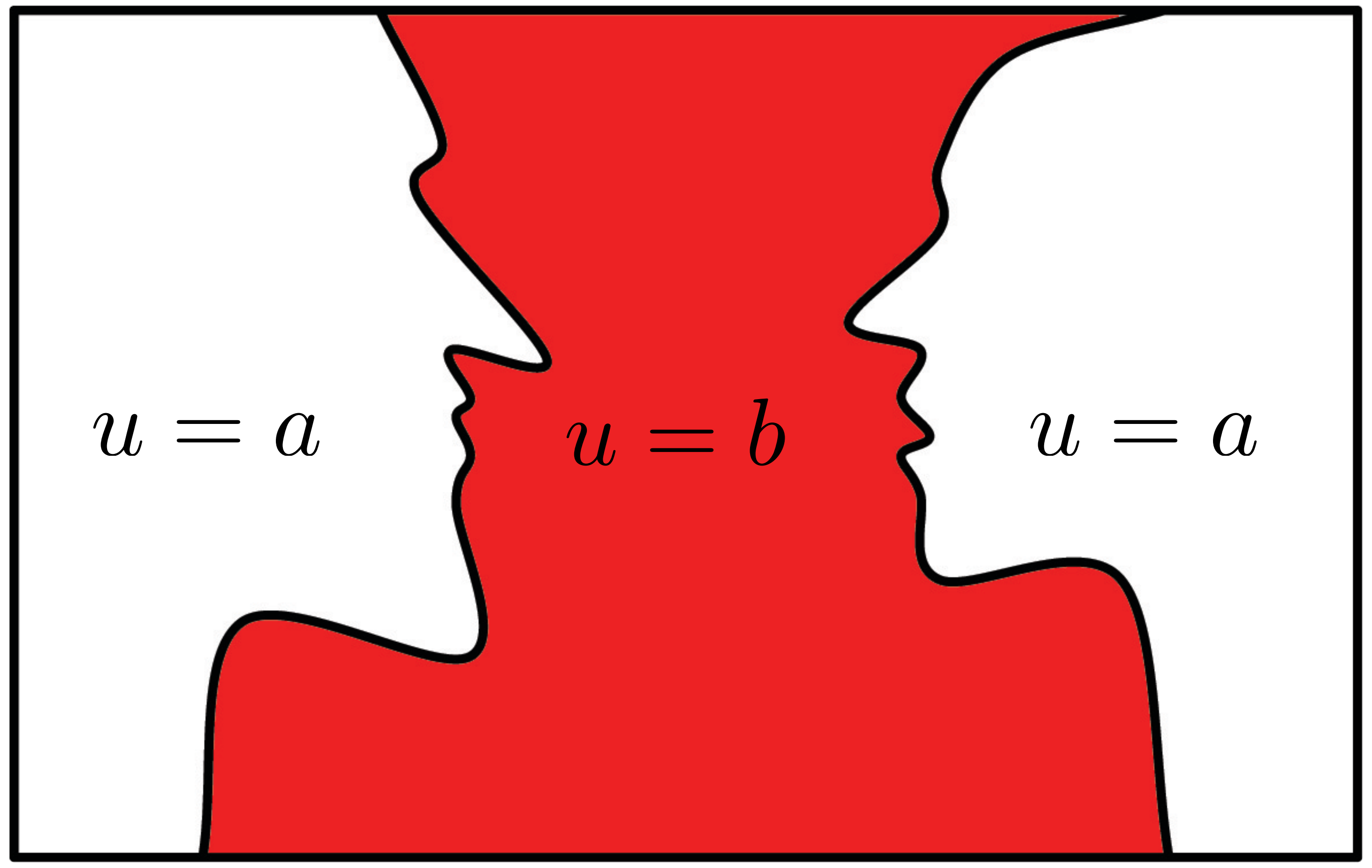
$$\min_{\substack{u \in L^1(\Omega) \\ \int_\Omega u \, dx = V}} \int_\Omega W(u) \, dx$$

$$a|\Omega| < V < b|\Omega|$$



W = Gibbs free energy density

classical theory for phase transitions for non-interacting fluid



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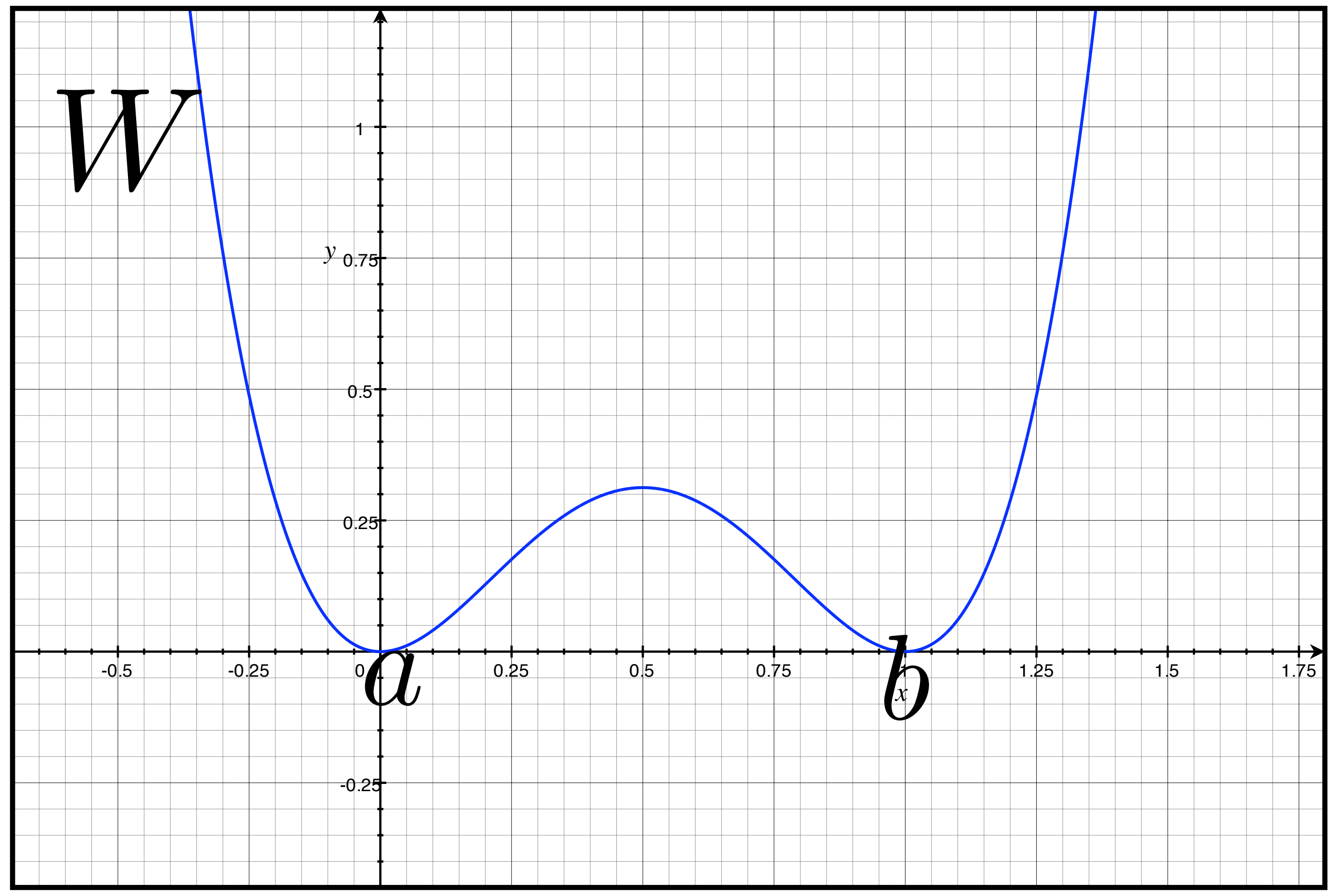

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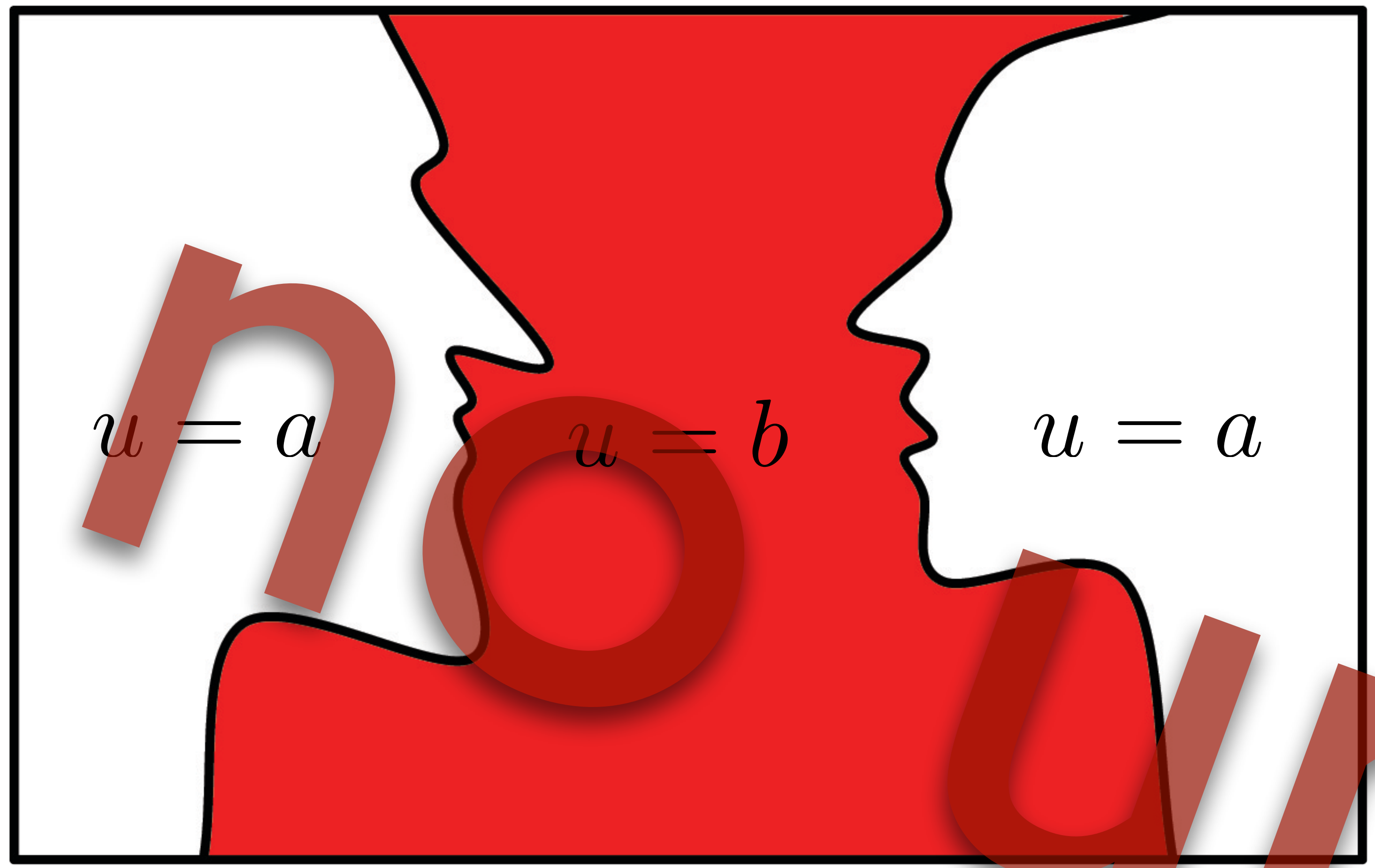

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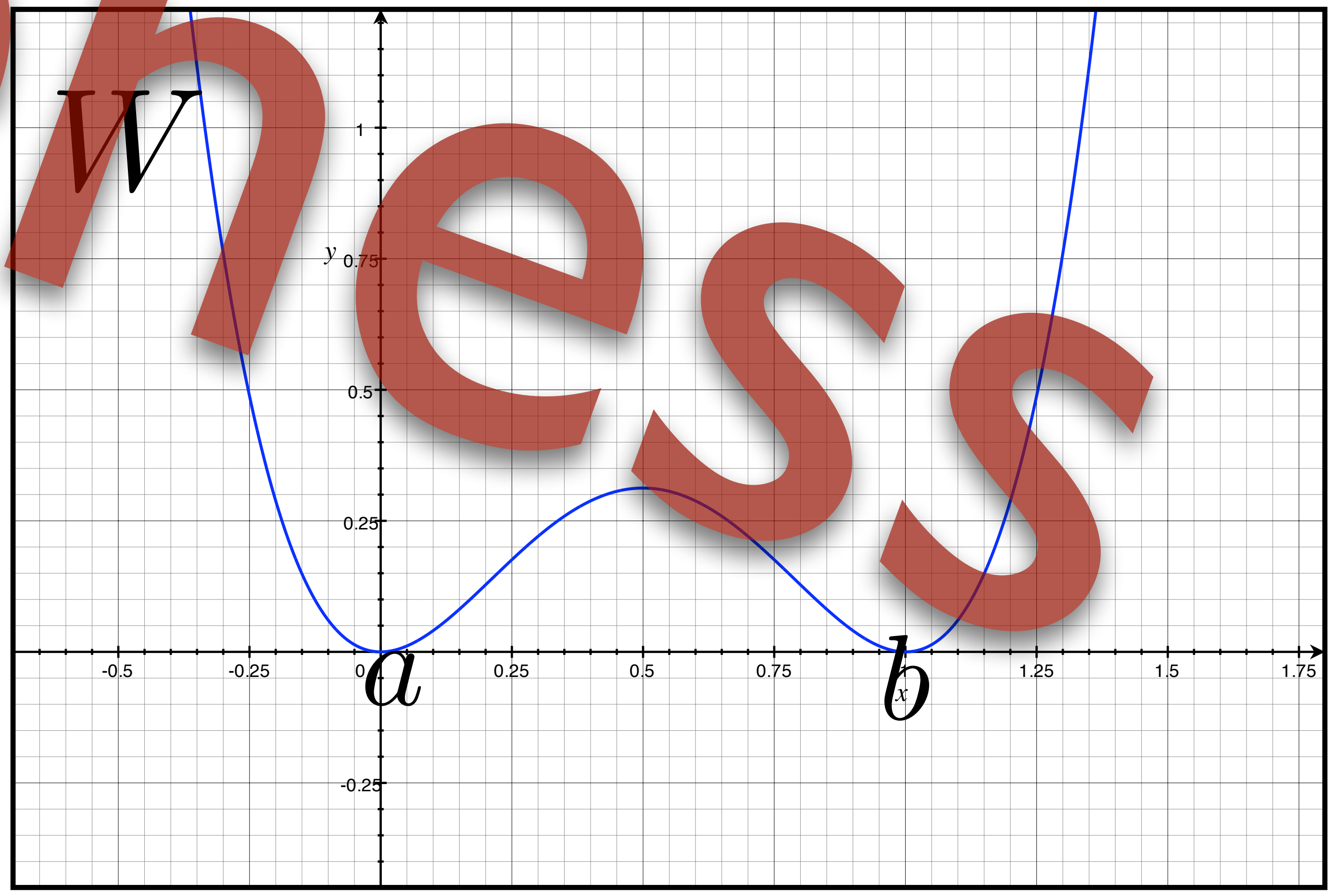

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no uniqueness

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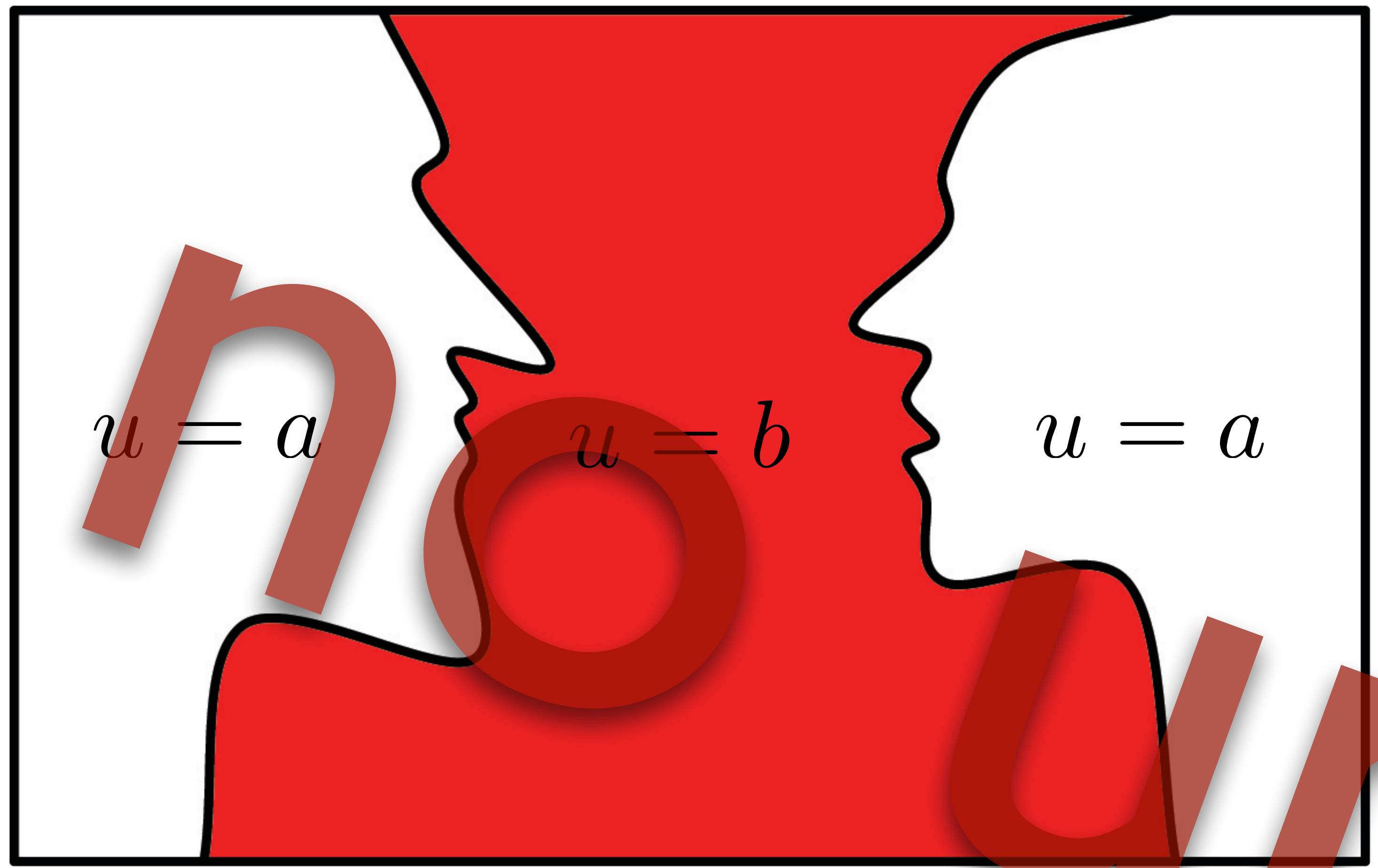

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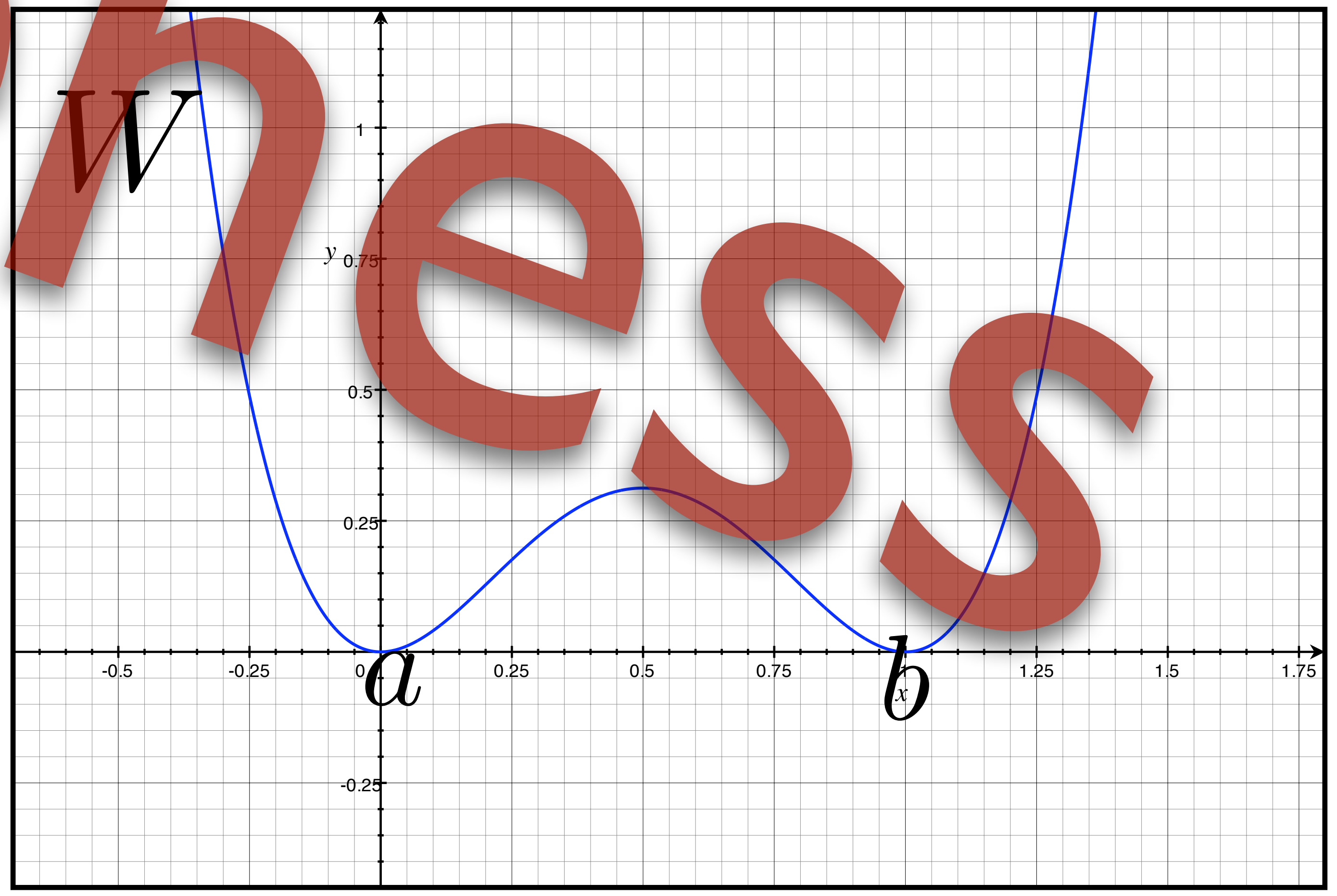

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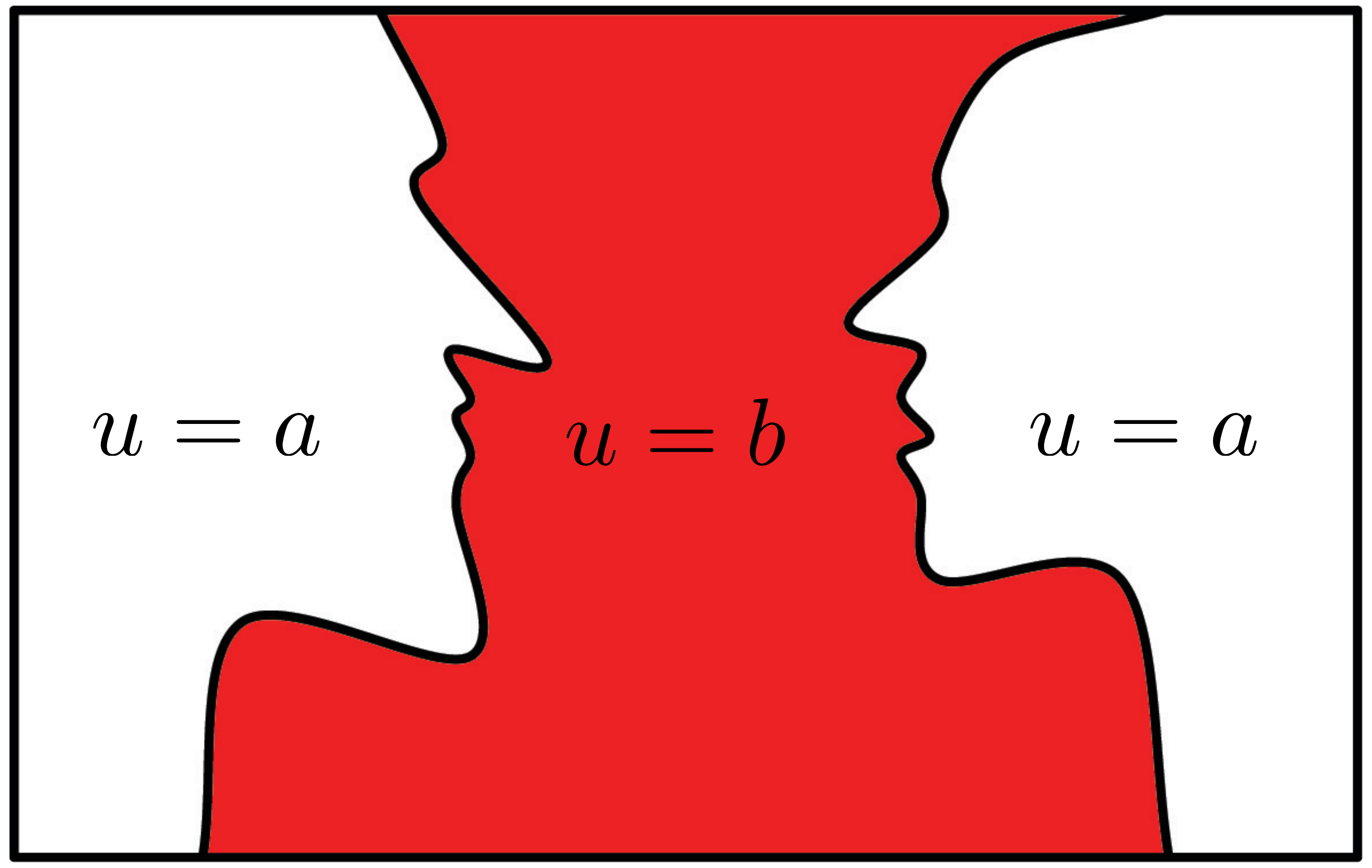

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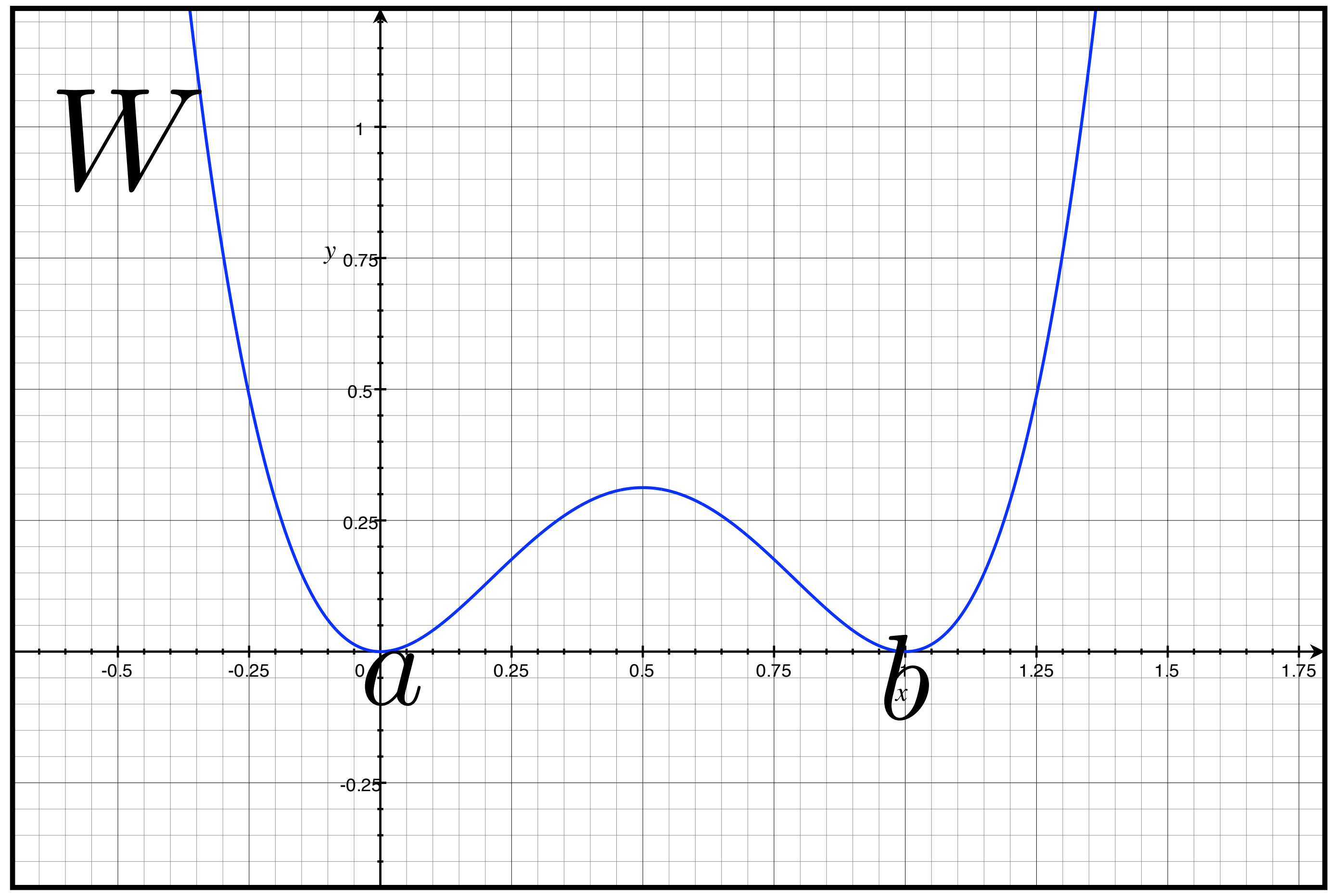

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Van der Waals – Cahn – Hilliard

+ perturbation ⇨ selection criterium

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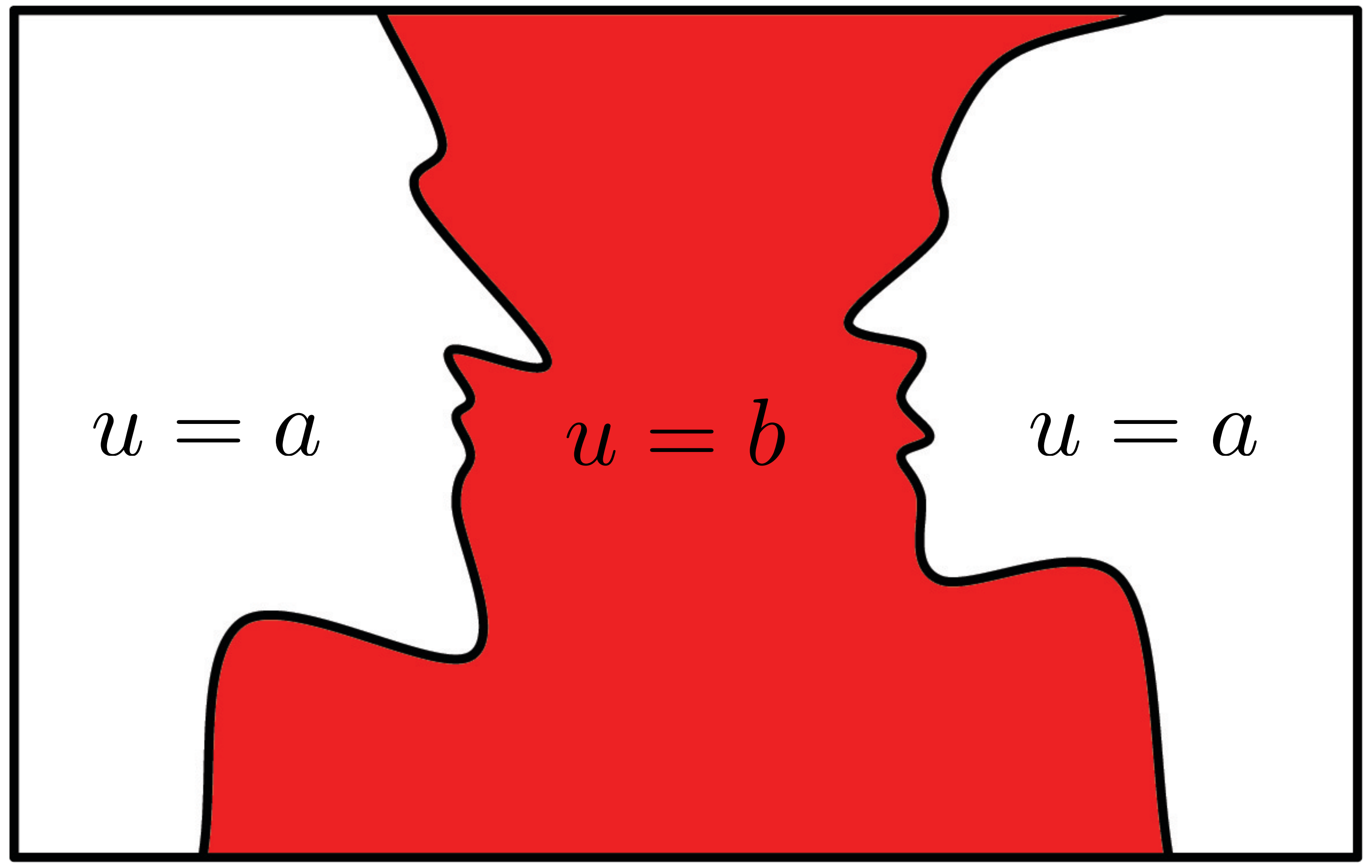

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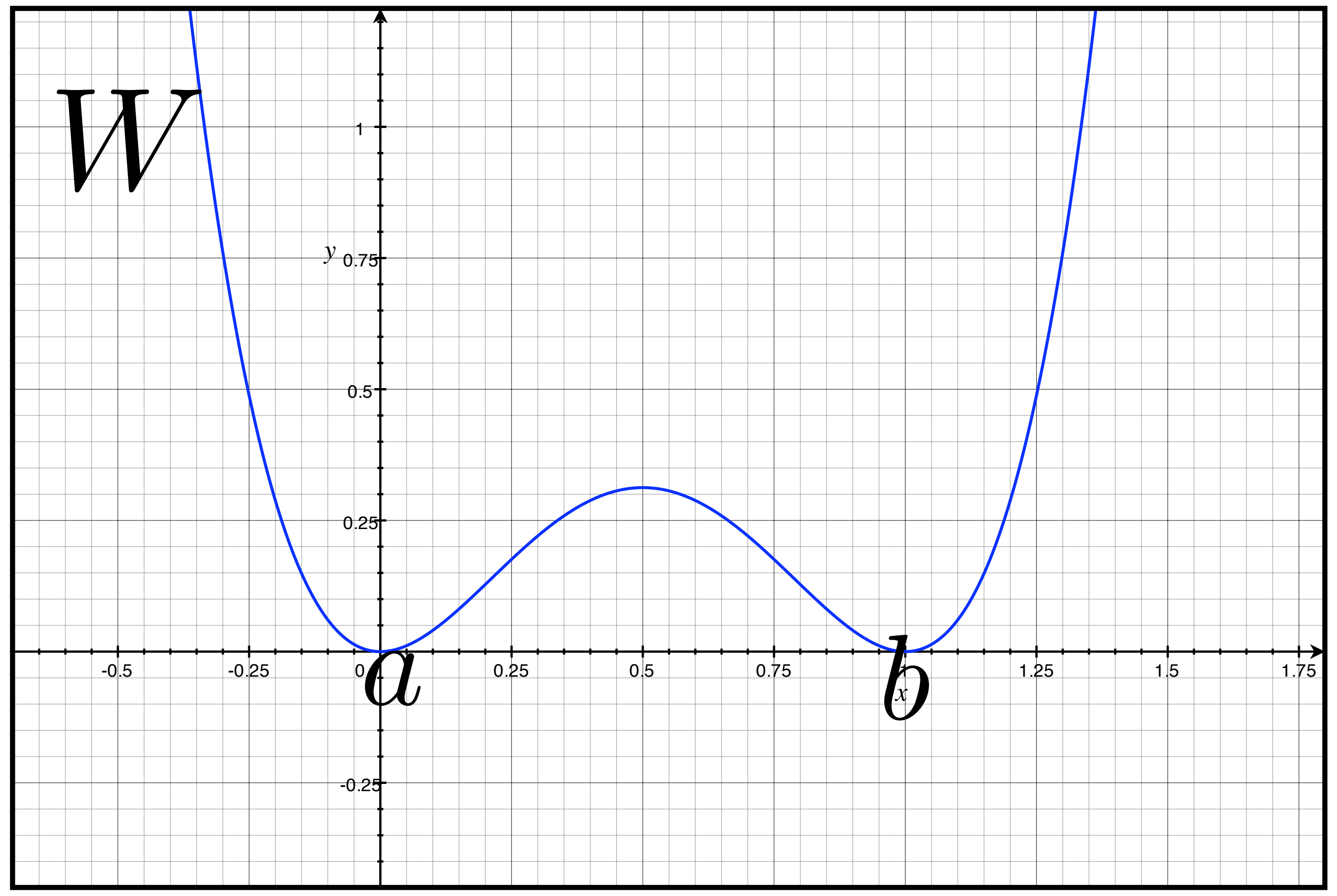

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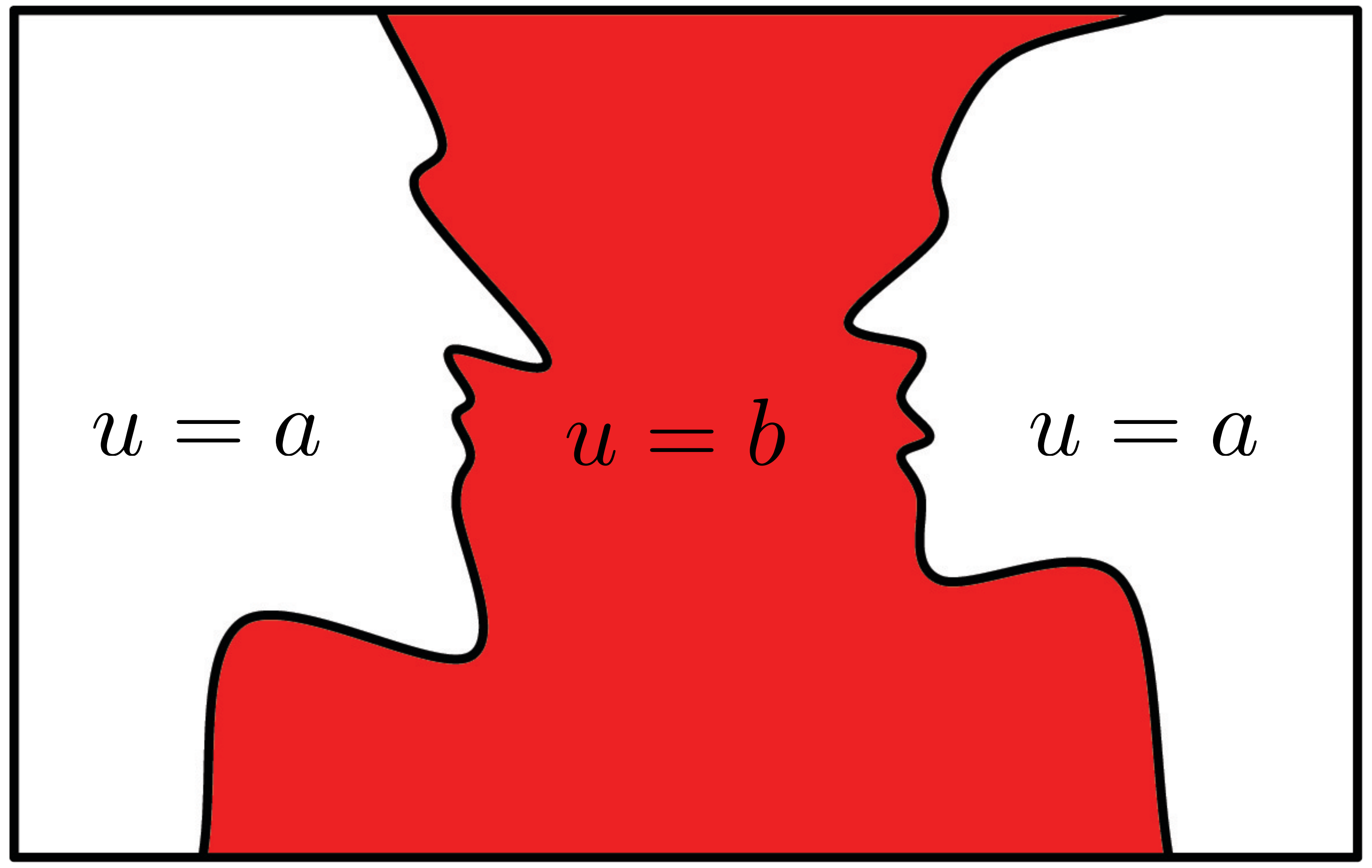

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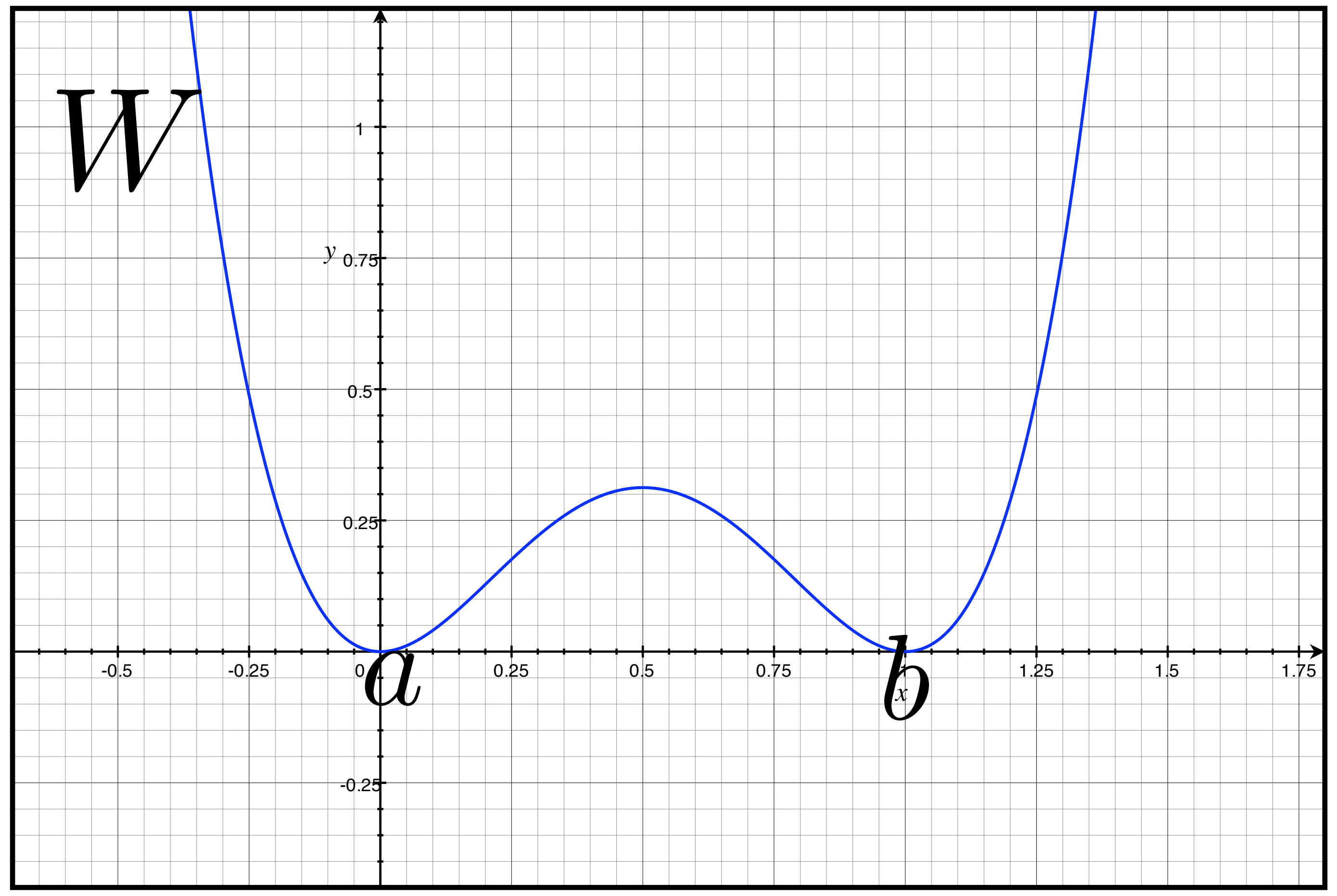

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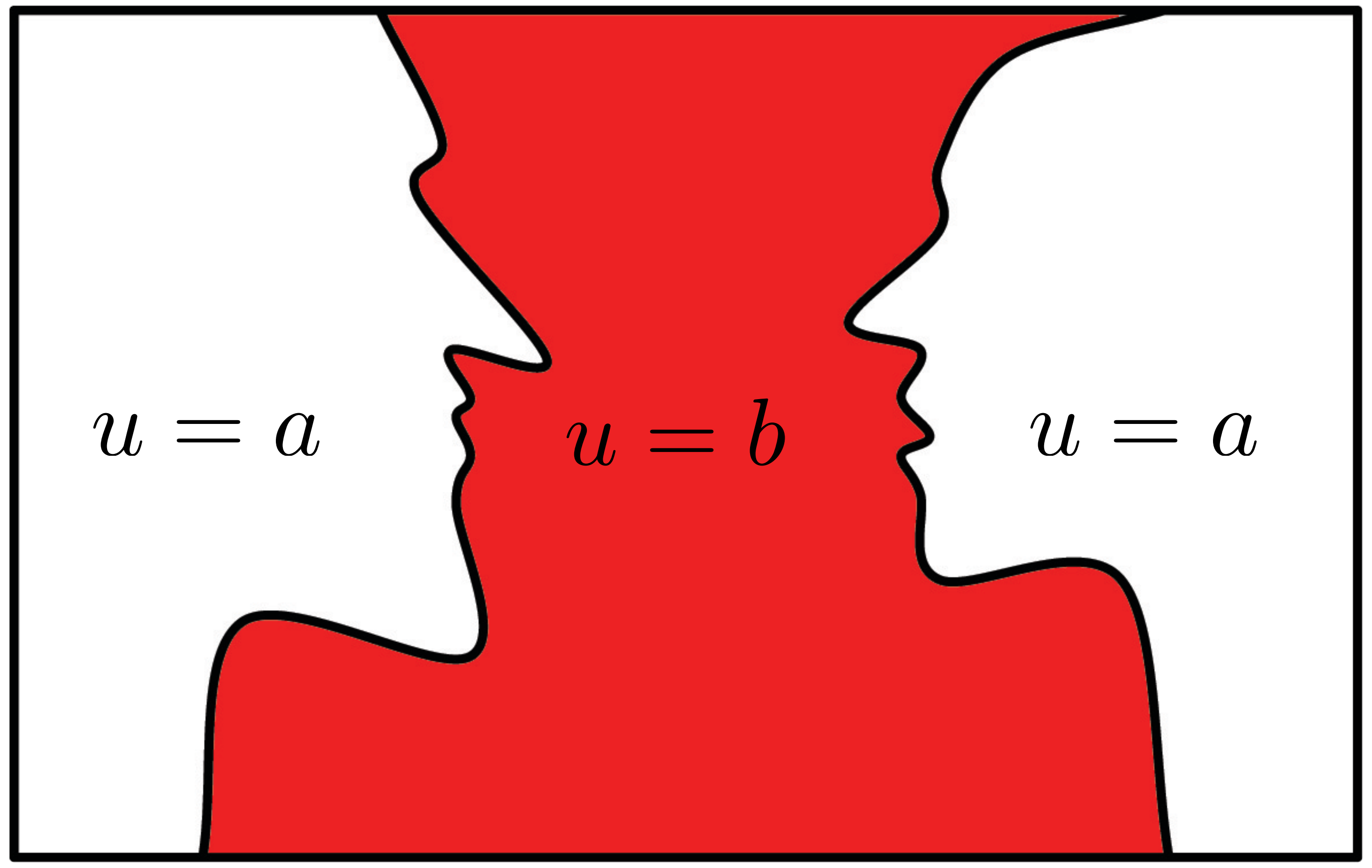

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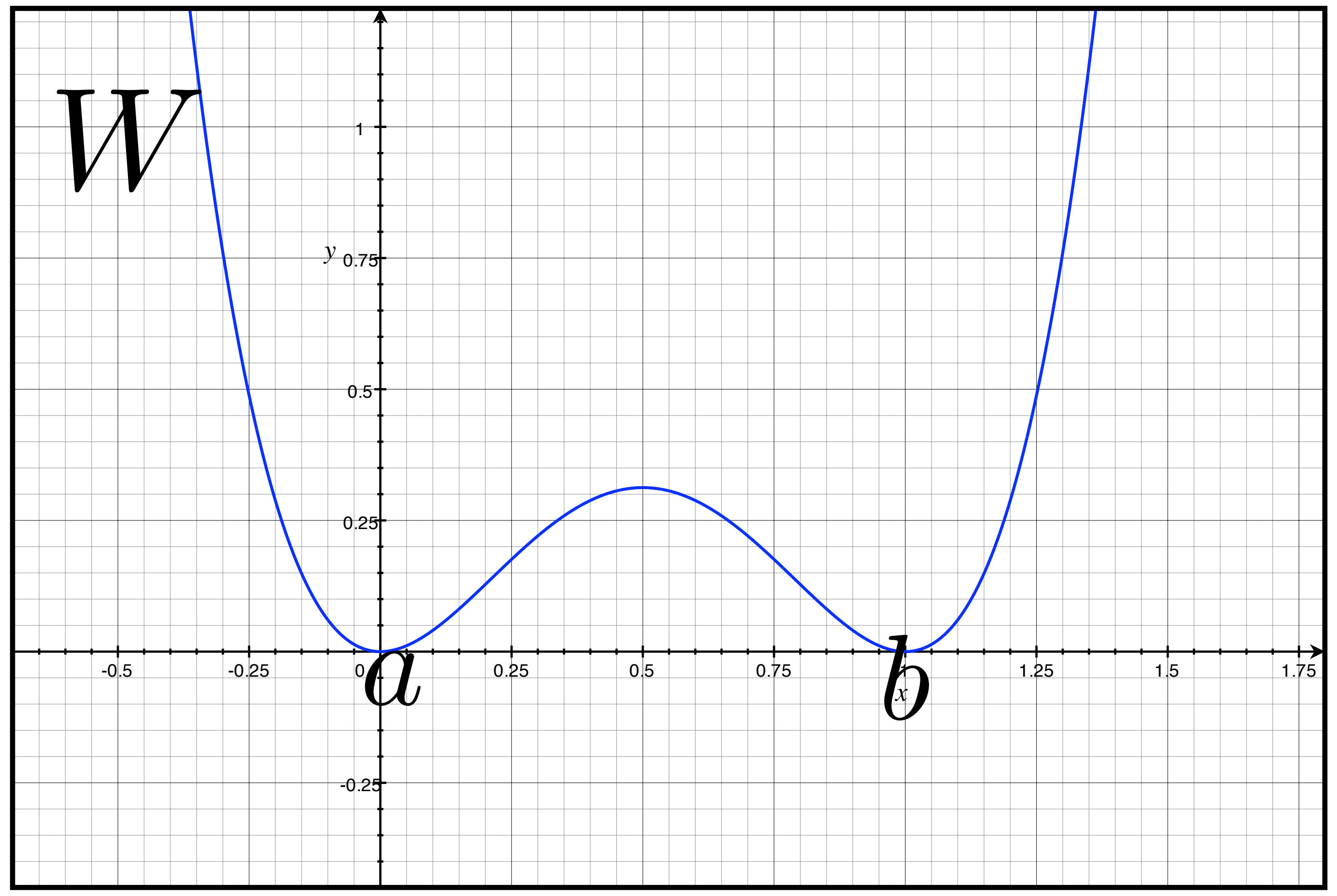

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Solutions u_ε

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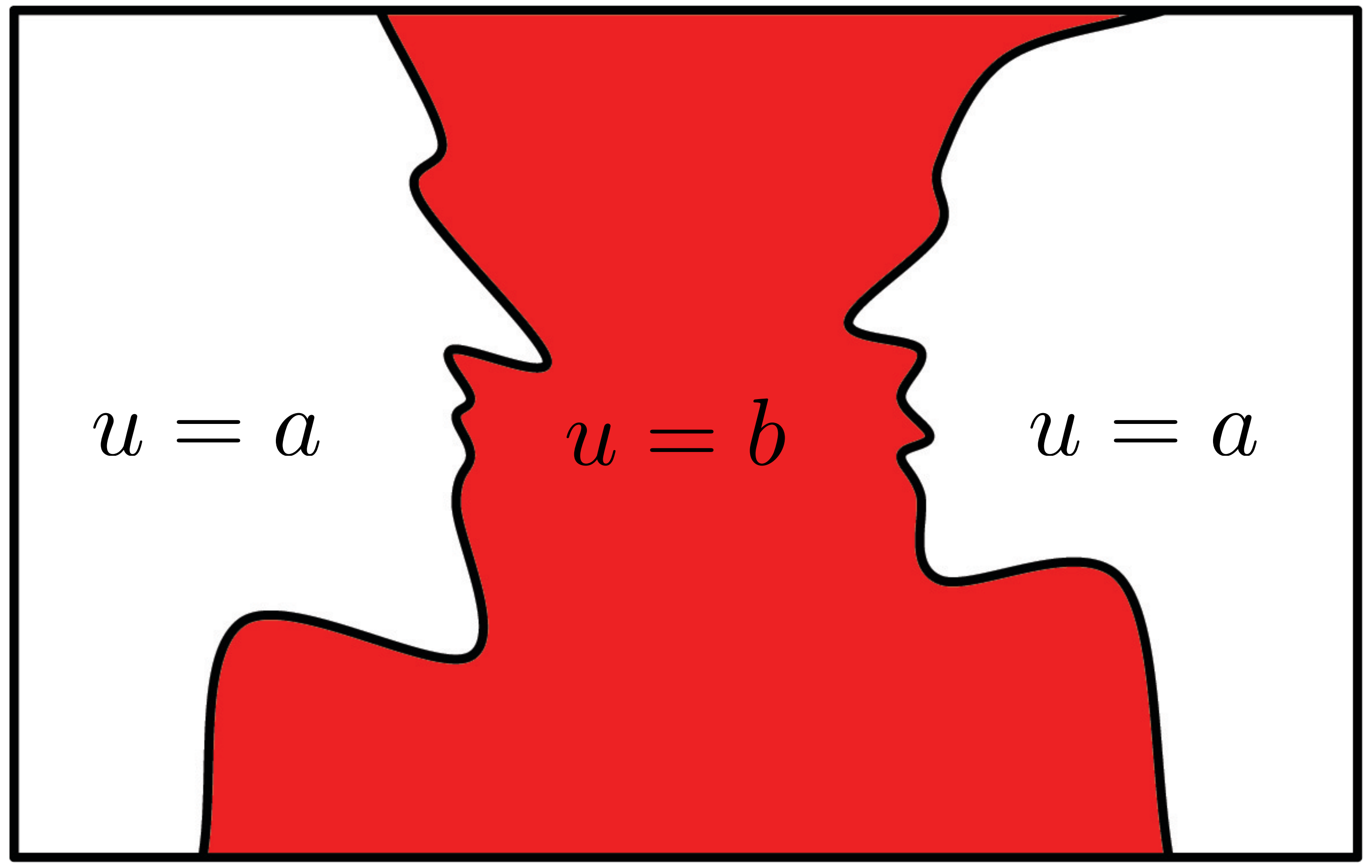

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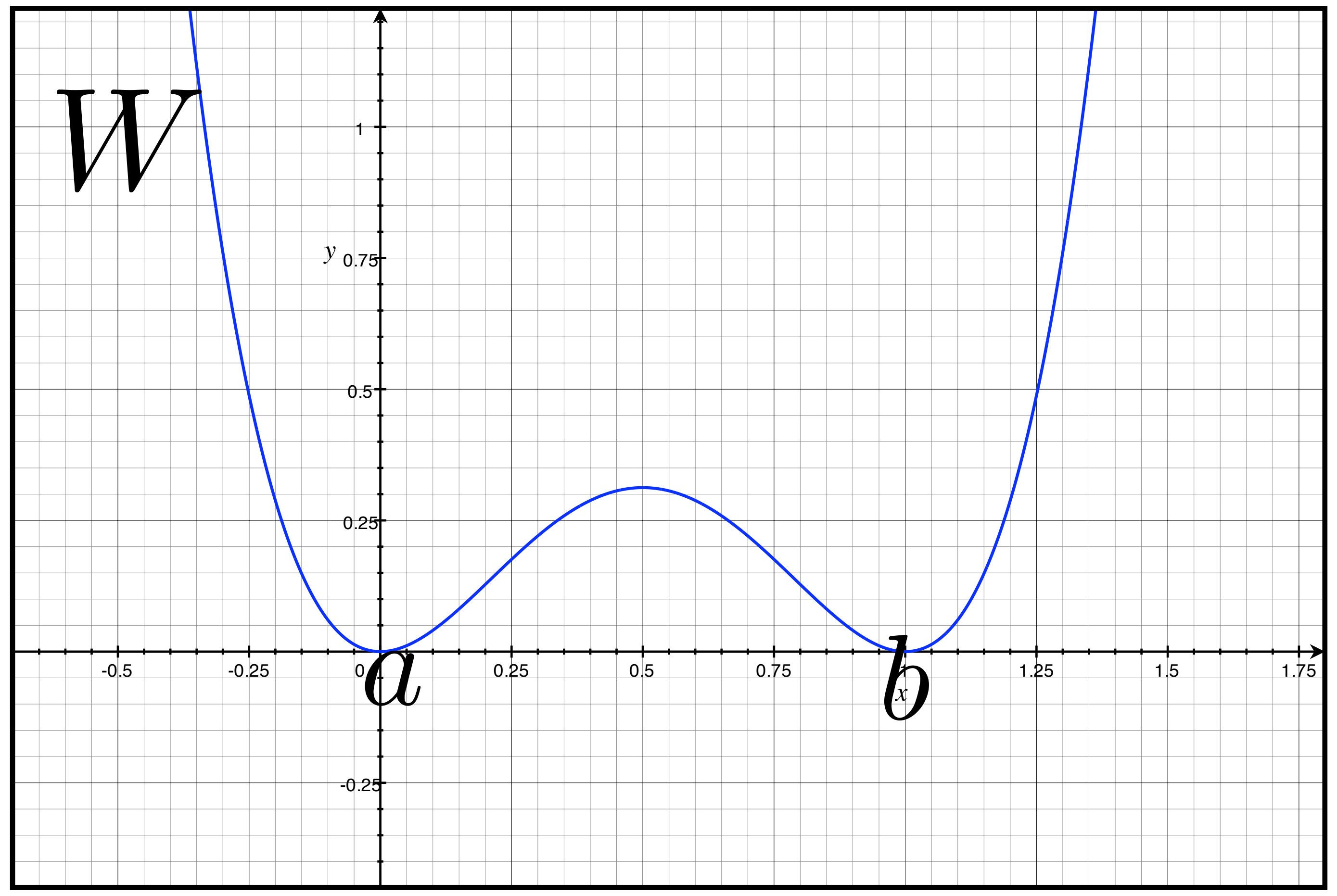

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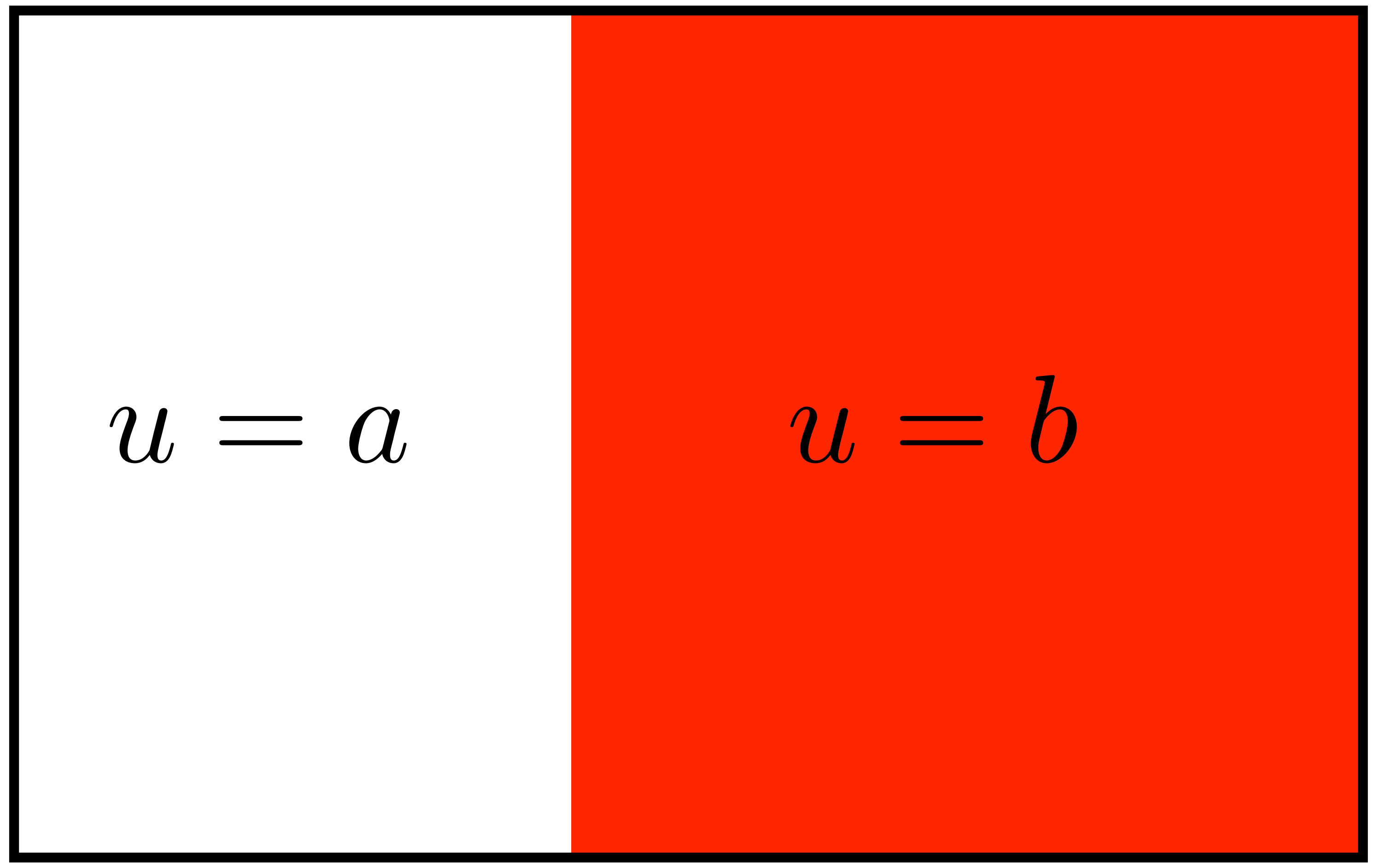

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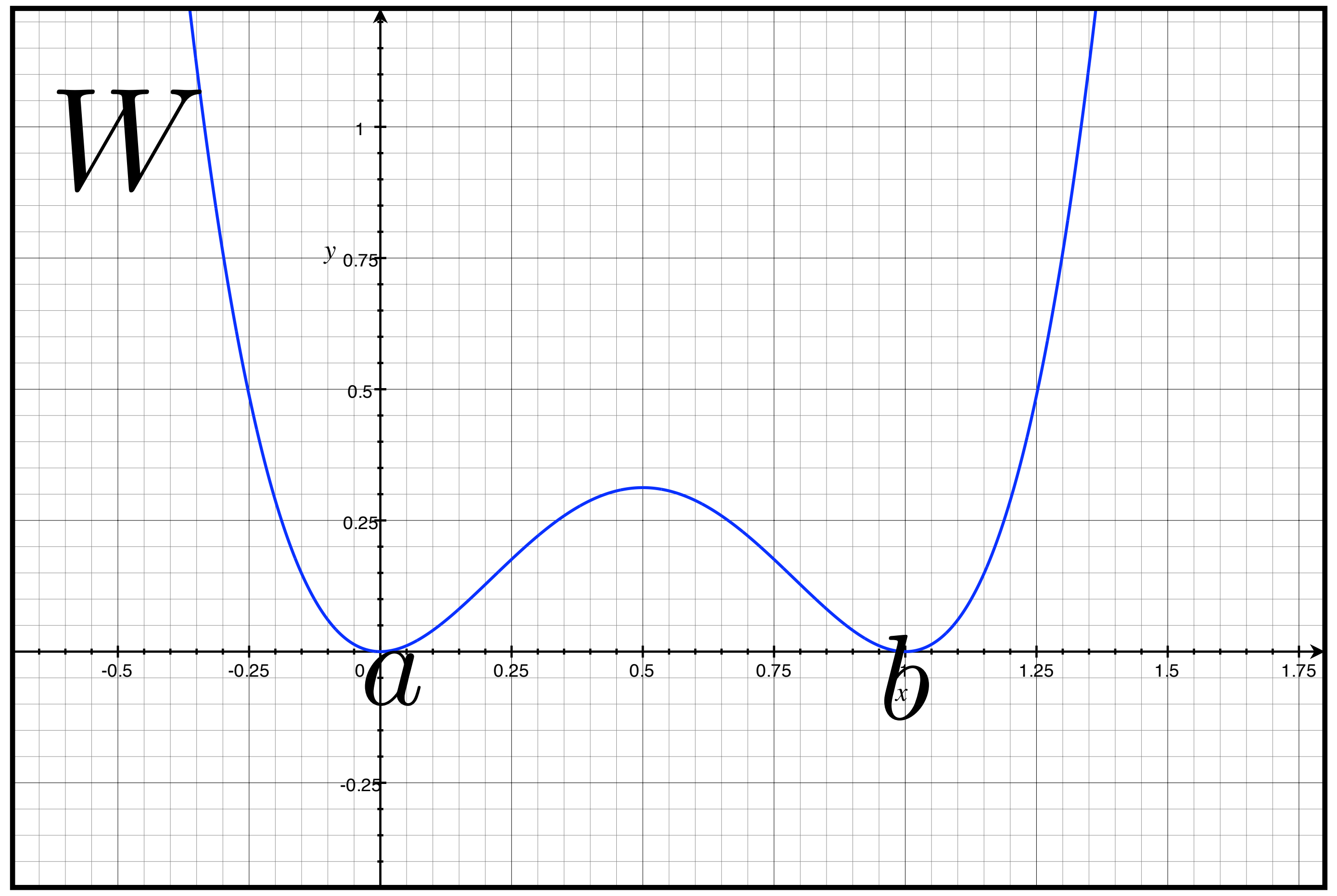

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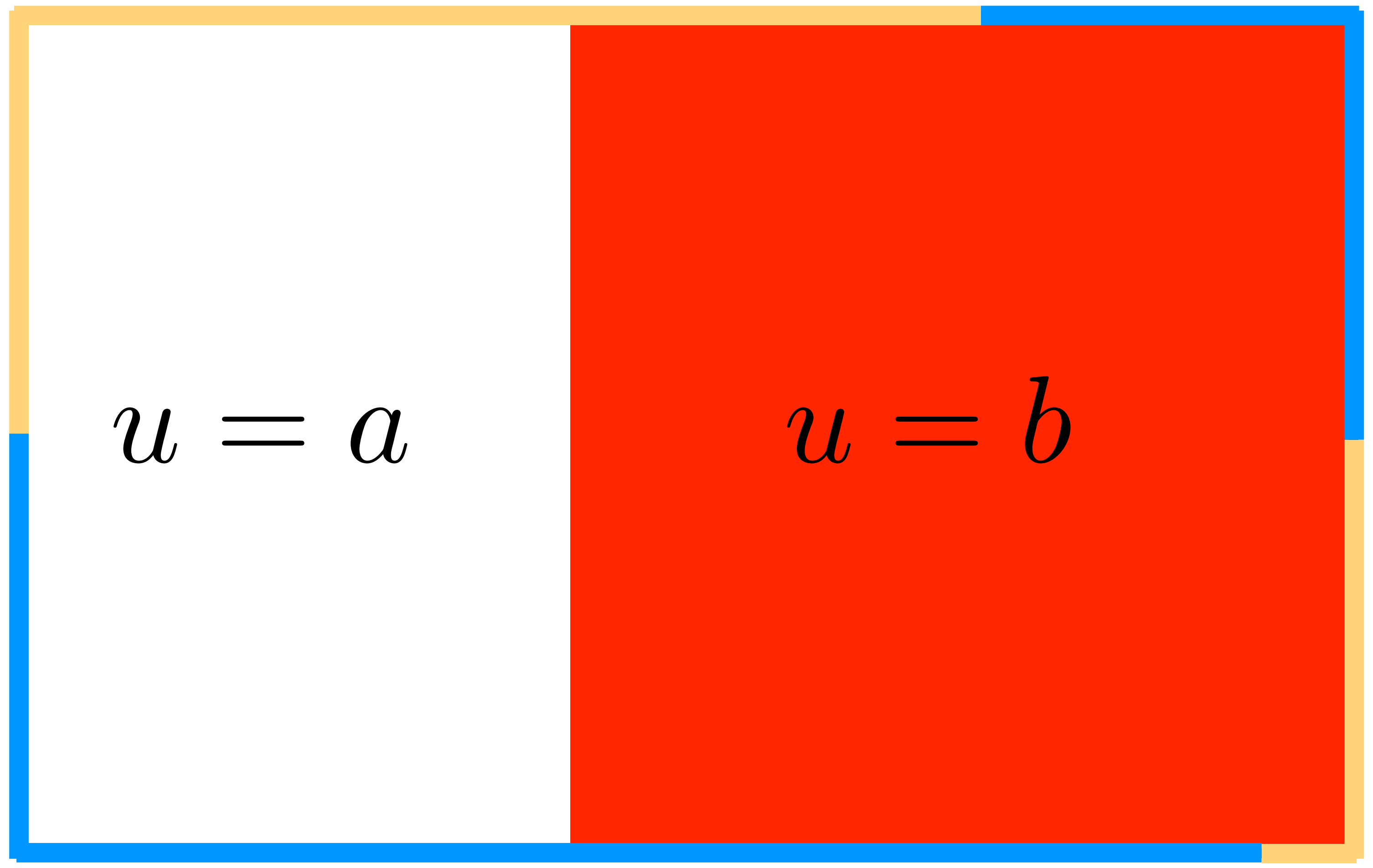

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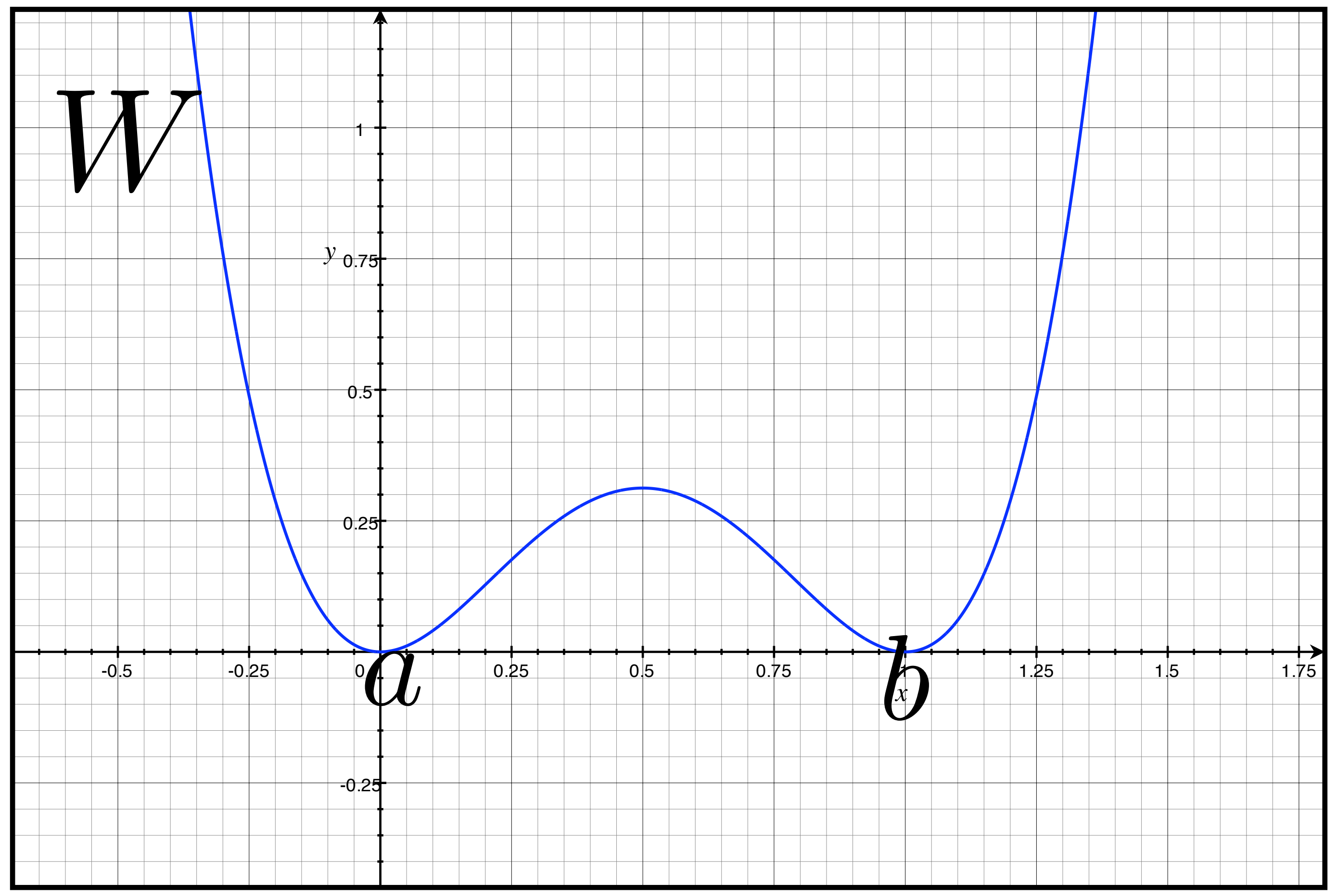

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