

An organized shuffle

Take a brand new deck of cards. If you do not have one, take an ordinary 52-card deck and sort the cards so that from the top face down, you go through all the spades in order, then all the hearts, then all the diamonds and finally all the clubs. Remove the jokers.

Now holding the deck face down, deal the cards alternately into two piles face up ($A\spadesuit$ to the left, $2\spadesuit$ to the right, $3\spadesuit$ to the left, and so on). When you have done this, put the right pile on top of the left, turn the pile over, and repeat this process several times. The aces of spades will always be the first card you deal out and the king of clubs will be the last.

This process will mix the cards up, and one would think that the more often you do this, the more mixed-up the cards become, so that they essentially become random. However, I suggest you do this and observe carefully what happens. It will not take long.

It turns out that when you repeat the process eight times, the order of the cards will be exactly the same as it was when you started.

Suppose that you renumber the cards from 0 to 51, so that initial order of the cards from the top when face down is the natural order of the cards. The evenly numbered cards will be dealt into the left pile and the oddly numbered ones into the right pile. When you put the right pile onto the left, the order of the cards will now be:

$$0, 2, 4, \dots, 50, 1, 3, \dots, 51.$$

After you repeat the process and reconstitute the deck, the order of the cards will be:

$$0, 4, 8, \dots, 48, 1, 5, \dots, 49, 2, 6, \dots, 50, 3, 7, \dots, 51.$$

The number of the card in the first position is always 0 and in the last position is always 51. The number of the card in the second position is, successively: 1, 2, 4, 8, 16, 32, 13, 26, 1. The number of the card in the third position is 2, 4, 8, 16, 32, 13, 26, 1, 2 and in the fourth position is 3, 6, 12, 24, 48, 45, 39, 27, 3. The number on the card in a given position gets doubled with each shuffle if it is less than 26. However, if it is 26 or more, the number on the card in the same position is its double minus 51. Thus, initially, the card in the 37th position after the first shuffle will be $2 \times 37 - 23$.

You might investigate what happens if you do the same procedure, except that each time you put the left pile on top of the right before

2

turning the deck over and dealing out. You will come back to the initial order, but how long will it take?