

Running it backwards

What is the value of the product: 3516×8274 ? Now write the digits of these two numbers backwards and calculate the product: 4728×6153 . It turns out that the two products are the same!

If you do not have a calculator handy, there is another way to check that the two products are equal. Every integers can be written as a product of powers of primes, so we can factor the numbers involved and see whether the two products decompose the same way. Here is what we have: $3516 = 4 \times 3 \times 293$; $8274 = 2 \times 7 \times 591$; $4728 = 8 \times 591$; $6153 = 3 \times 7 \times 293$. Both products are equal to $8 \times 3 \times 7 \times 293 \times 591$. (It is interesting that the pairs of numbers 3516 and 6153 have such a large factor in common, as do the pairs 8274 and 4728. It makes you wonder what is the largest common factor possible for two four digit numbers, one the reverse of the other.)

A more marked example of the same phenomenon is this product:

$$992 \times 483 \times 156 = 651 \times 384 \times 299 = 2^7 \times 3^2 \times 7 \times 13 \times 23 \times 31.$$

I am not sure how one finds such oddities in the first place, but some reader with a lively computer might be able to generate some more. However, it is not too hard to generate pairs of numbers, each with two digits, such that their product is the same as the product of their palindromes (numbers written backwards).

I will also leave the reader with the task of generating a pair of numbers, one with two and one with three digits with the same property. As a hint, if you make the digits small enough, you do not have to contend with carries.