The Rule of Eleven

Imagine that you, South, are a bridge player. You are one of four players, the other three in clockwise order are West, North and East. Each of you have been dealt thirteen cards from an ordinary deck; you and North are partners in competition with partners East and West. You are faced with the task of taking eight tricks with no trump. The play, similar to that of hearts and whist, is that one of the players leads by placing a card face-up on the table. The other players follow in clockwise order placing a card that must be of the same suit if they possess it. The highest ranking card of the suit led (ace being high) takes the trick, and its winner has the next lead.

The *contract*, two no trump (*i.e.* two plus six tricks to be won), has been arrived at by a process of bidding, with South delivering the final bid. West leads the first card. At this point, North lays his hand (the "dummy") on the table so that all players can see it, and for the rest of the hand, North's cards will be played by South.

A characteristic of bridge is that through the bidding and the play of cards, there are legitimate conventions for passing information to your partner, but you have to remember that your opponents are "listening in". One of these conventions is that, when faced with a no trump contract by South, West leads the fourth highest of the longest and strongest suit in his hand. The purpose of the lead is to convey to East that West may have a suit that is capable of taking tricks, and that if East ever gets the lead, he should lead back West's opening suit. You often lead a lower card in a suit to encourage your opponent to play a high card which may fall under an even higher card from your partner.

Of course, South is aware of this convention, and does a quick calculation. South subtracts the rank of the suit led from eleven. This tells you the number of cards in that suit higher than the West's lead that are in the three hands other West's. Since South knows what is in his own hand and in dummy's, he can deduce how many cards higher than the lead are in East's hand. This is an application of the *Rule of Eleven*.

For example, suppose that West leads the seven of spades. South has the ten and jack of spades in his hand, and he sees the queen of spades in the dummy. By the Rule of Eleven ($11 - 7 = 4$), South realizes that East has one spade greater than the seven, and deduces that West must be holding three of the Ace, King, nine and eight of spades. East must have one of these cards, and if it is played on the first trick, then

South knows exactly the three cards greater than the seven that West started with.

The justification of the convention involves a bit of mathematical reasoning. You may have your own way of envisaging the situation, but here is one possibility. The rank order of cards from the top are Ace, King, Queen, Jack, 10, 9, etc.. Thinking of the top four cards as 14, 13, 12, 11, we see that there are $14 - 7 = 7$ cards greater than the seven. Since West led the fourth highest spade, three of these seven cards are in West's hand. The other four must be elsewhere. We can think of 4 as

$$7 - 3 = (14 - 7) - 3 = (14 - 3) - 7 = 11 - 7,$$

thus verifying the Rule of Eleven.

Note that West is not forced follow this convention with his lead and may for good reason lead in some other way. Often East (and South) will realize this, and act accordingly. At times, West may be trying to deceive the opponents. So whether or not the convention is honoured on any occasion becomes a matter of judgment.

Back in the days when more students played bridge than is now the case, I used to compare mathematics to bridge. There are first of all the rules of the game that must be adhered to; these concern the dealing of the cards, the bidding and the playing and winning of tricks. On top of this there are certain conventions, ways of proceeding under normal circumstances that are not imposed by the rules. Then there is a technical discipline, developing a good memory and keeping track of the play so that you know how many cards (and which ones) of each suit have been played. There is reasoning and analysis. You come to conclusions from the bidding and the play of the cards, so that, for example, you can surmise the distribution of the suits in the opponents' hands. Part of this reasoning is hypothetical, considering possible situations that might occur and the outcomes of certain actions. In particular, since the dealing of the cards is random, one needs to estimate the probability of certain configurations and act according to the odds. A good bridge player has an overview of the hand and devises a strategy to gain the contract. Finally, there is creativity. The master bridge player often brings an unusual perspective to bear and so chooses a play outside of the normal options that most players would be bound by.

These are ingredients of mathematics: the foundational rules, the conventional ways of proceeding, the memory work, the discipline of

working correctly and efficiency, the reasoning and analysis, the strategizing and finally the creativity. Mathematics is not just blindly memorizing and applying formulae, but it embraces all of these aspects. People often have a fear of mathematics that is unwarranted; in their work or recreation, they may be involved in things that are equally or more challenging mentally than any mathematics they will meet at school.