The convenience store

Sometimes a problem arises that makes you wonder how anyone ever thought of it. The *Seven-Eleven* problem that appeared on a Swedish mathematical competition some years ago is one of these. A patron walks into a Seven-Eleven store and makes four purchases. The salesclerk pulls out his pocket calculator, computes the total cost and announce that $7.11 has to be paid.

However, the patron notices, to his horror, that instead of adding the prices together, the clerk *multiplied* them. When he pointed this out, the clerk apologized and recalculated, adding the four amounts this time. The answer was again $7.11. What did each of the four items cost?

If you want to wrestle with this for a while, do not proceed beyond the starry curtain until you want some support.

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Expressing the price in dollars and cents involves a decimal point, which is a complicating factor. So let us begin by expressing the prices in cents alone; let them be a, b, c and d. These are whole numbers that satisfy the two equations

$$a + b + c + d = 711$$

and

$$(a/100) \times (b/100) \times (c/100) \times (d/100) = (711/100).$$

The last equation can be rewritten as

$$a \times b \times c \times d = 711 \times 1000000.$$

Every whole number can be factored as a product of primes. For example, 711 is the product of 3, 3 and 79. The right side of the last equation can be rewritten as the product

$$2^6 \times 5^6 \times 3^2 \times 79.$$

Since 79 is a prime, it must divide one of the numbers a, b, c and d. Suppose it divides a. Then a must be one of the available multiples of 79: 79, 158, 237, 316, 395, 474, 553, 632.

Since not all of a, b, c and d can be multiples of 5 (otherwise their sum would be), either three of them are divisible by 25 or at least one of them is divisible by 125. If three of them are divisible by 25, they must be b, c, d, as a is too small to be divisible by both 25 and 79. But then $b + c + d$ is a multiple of 25; you can check that it is not possible to write 711 as the sum of a multiple of 79 and a multiple of 25. Therefore, one of the numbers, say b. must be a multiple of 125. This should narrow down the search for you to proceed apace. Solvers are welcome to submit their solutions.