

This column is for your mathematical amusement. Its author is very happy to correspond with readers about mathematical matters, and hopes that the column will turn out to be a dialogue with readers of the Frontenac News. His email address is barbeau@math.utoronto.ca.

The number 142857

The number 142857 looks like any common garden variety six-digit number. However, it holds an interesting secret. Multiply this number by 2 and you get 285714. This product has the same six digits and, if you imagine them written in a circle, they are in the same order. There is more:

If you multiply 142857 by 3, 4, 5, and 6, you get in turn these numbers 428571, 571428, 714285, 857142. Is this just a fluke? Or is there something interesting going on here? We get a clue if we go ahead and now multiply it by 7 to get the product 999999.

There are two significant features of this situation. First, 7 is a prime number, that is a number that is divisible only by 1 and itself. Other prime numbers are 2, 3, 5, 11, 13, 17, 19, 23, and so on. Secondly, this prime number divides evenly into a number all of whose digits are 9s.

Let us look at another case. Start with the prime 13, which divides evenly into 999999. In fact, 999999 divided by 13 gives the quotient 076923. Even though it is not customary to take 0 as the first digit of a number, you will see why this is done in a minute. Now multiply this number by 2, 3, 4, and everything up to 12: you a list of multiples that start with 153846, 230769, 307692. Now it is in your hands; pull out your pocket calculator and see why the set of multiples is interesting.

Now you can make some discoveries of your own. We will ignore the primes 2 and 5 because they are involved in the base 10 of numeration. But look at every other prime and see if you can get some patterns. The prime 3 divides into 9; 11 divides into 99; 37 divides into 999; 17 divides into a number that has sixteen digits, all nines. Look at the multiples of the quotients after you divide by the prime.