Problems on Patterns for elementary teachers and their pupils

P.1. Here are three numerical equations. Extend the list to include some more correct equations following the same idea. How would you verify that the equations you obtain actually hold?

$$3^2 + 4^2 = 5^2$$
$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$
$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2$$

P.2. Discuss how the trio of sequences below might be continued and determine as many patterns involving them as you can. In particular, look for ways to construct Pythagorean triples (three integers for which the square of the largest is equal to the sum of the squares of the other two).

First soprano	Second soprano	Contralto
0	1	0
1	1	1
2	3	6
5	7	35
12	17	204
29	41	1189

P.3. Find as many sets of four numbers as you can for which the cube of the largest number is the sum of the cubes of the three smaller numbers. Here are a few examples to help

you out:

$$3^3 + 4^3 + 5^3 = 6^3$$

$$12^3 + 19^3 + 53^3 = 54^3$$

$$27^3 + 46^3 + 197^3 = 198^3$$

$$48^3 + 85^3 + 491^3 = 492^3$$

$$3^3 + 10^3 + 18^3 = 19^3$$

$$12^3 + 31^3 + 102^3 = 103^3$$

$$27^3 + 64^3 + 306^3 = 307^3$$

$$48^3 + 109^3 + 684^3 = m685^3$$

$$9^3 + 12^3 + 15^3 = 18^3$$

$$28^3 + 53^3 + 75^3 = 84^3$$

$$65^3 + 127^3 + 248^3 = 260^3$$

$$4^3 + 5^3 + 3^3 = 6^3$$

$$16^3 + 23^3 + 41^3 = 44^3$$

$$64^3 + 107^3 + 405^3 = 408^3$$

$$4^3 + 17^3 + 22^3 = 25^3$$

$$16^3 + 47^3 + 108^3 = 111^3$$

$$64^3 + 155^3 + 664^3 = 667^3$$

Be sure to check the accuracy of this equations as well as other equations that you find. Try to arrange that the greatest common divisor of the four numbers in each of your sets is 1 (why do I ask that)?