Icons of Mathematics: An Exploration of Twenty Key Images
By Claudi Alsina & Roger B. Nelson
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The author of a geometry book directed at a general audience has a daunting task. For about 2500 years, amateur and professional mathematicians in Europe and Asia have uncovered an abunadance of fascinating results about simple geometric figures, particularly circles and triangles, and the end is apparently not in sight. Many different techniques have developed to establish them, each shining its own particular light and allowing for different insights and connections to be made. How can one choose among such a wealth of facts and arguments and arrive at a book that is readable, representative, focussed and compact?

The authors of the book under review use twenty diagrams, or "icons", as an organizing principle. Many of them have a particular historical or cultural significance, such as the Yin-Yang symbol (a circle bisected by two semi-circles of half the diameter), the "windmill" diagram used by Euclid in his proof of the Pythagoras theorem (I:47) (a right triangle with squares constructed outwards on the sides), the three circles of a standard Venn diagram, and a trapezoid figure used by President Garfield in his proof of the Pythagoras theorem. Others are just representative figures to introduce a theme.

The prerequisites for this book are modest. The reader should have a secondary background in Euclidean geometry and trigonometry, along with an active imagination. Most of the results are proved in an informal way; the chief tools are standard Euclidean arguments, algebraic manipulation and proofs without words that involve dissections and shifting figures around. A small number of propositions are mentioned without proof, and some are listed as challenges at the end of each chapter, with solutions provided in an appendix. Each chapter is lightened by digressions on mathematical personalities and artefacts, history and background, and occurrences of geometrical figures in ordinary life. For example, on page 203, we find photographs of star-shaped badges with 5, 6, 7 and 8 points worn by law enforcement officers along with speculation about their origin. Two pages later, we learn about the role of star polygons in the design of columns for a modern church in Barcelona by Antoni Gaudi (1852-1926).

This book need not be read straight through. Since the twenty chapters are generally self-contained, readers can forage at random and follow something that catches their fancy. *Crux* subscribers will find a lot of familiar material along with results and arguments that will be new to them. The scope of the book can be suggested by a list of some topics that make an appearance. Apart from standard material on triangles, circles and inequalities, the authors touch on Dido's isoperimetric problem, Euclid's construction of the regular solids, reptiles, cevians, the butterfly theorem, conic sections, Reuleux polygons, star polygons, self-similarity, spirals, the Monge sponge and Sierpinski carpet and tilings. The treatment is light but satisfying, and readers wanting more are directed to other resources.

There are some themes that recur throughout the book, such as tilings and dissections, inscribed and escribed figures and cevians. The most prominent of these is the Pythagorean theorem, which is treated in many places. In the opening chapter, Euclid's diagram is generalized to the Vecten configuration (squares escribed on an arbitrary triangle) which leads to an insightful proof of the Cosine Law and the solution to two problems by the American puzzler, Sam Loyd (1841-1911). The second chapter uses the icon of one square inscribed inside another for an ancient Chinese proof of the Pythagorean theorem and continues to get a diagrammatic proof of standard inequalities. The trapezoidal diagram used by US President Garfield (1831-1881) to prove the theorem is pressed into service for some trigonometric equations. The chapter on similar figures provides the occasion to prove the Pythagorean as well as the Reciprocal Pythagorean theorem (that, if $a^2 + b^2 = c^2$, then $(1/a)^2 + (1/b)^2 = (1/h)^2$ where h is the altitude to the hypotenuse). The right triangle has a chapter of its own, in which Pythagorean triples are characterized and the Pythagorean relation is used to derive some inequalities. Another characterization of Pythagorean triples is found using two overlapping squares of areas a^2 and b^2 inside a square of area $c^2 = a^2 + b^2$. An arrangement of tatami rectangular mats in a square is behind a proof of the Pythagorean theorem credited to Bhaskara (1114-1185).

In the final chapter, infinitely many demonstrations of the theorem can be had by laying a hypotenuse grid atop a plane tiling of two different squares.

This is an enjoyable and useful book that provides a lot of material that could be presented to secondary students. The challenges at the end of the chapter are generally accessible and often require some insight to obtain an elegant solution. I found very little to quibble with. The approach to the proof of the result on page 133 (the locus of the centres of circles touching an ellipse and passing through a point inside the ellipse) seems backwards when a direct assault is just as easy. In Challenge 3.8, the diagram needs to be justified by the result that if $ABCD$ is a trapezoid with AB and DC both perpendicular to BC, E is the midpoint of BC and $\angle AED = 90°$, then ED bisects angle ADC.

This book suggests how geometry might be rehabilitated in the school curriculum. Now that it is no longer seen as the centrepiece of the syllabus and a prerequisite for later mathematics, it can be taught as an important milestone in human intellectual history, a celebration of human ingenuity, a more natural approach to proof and a source of personal pleasure. This would allow for more flexibility, and this book be a fine vehicle to carry such a course.