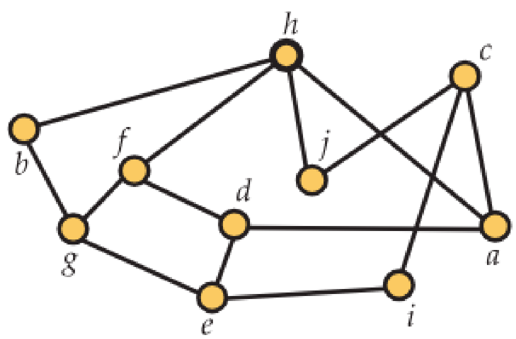
MAT344 Problem Set 6
(due 1pm July 9)

Note: For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.

Part A

In these problems, you will *analyze a counting problem by proving an exact or approximate enumeration*. Two of these problems will be marked.

Problem 1. Find the chromatic number, $\chi(G)$, of the following graph G, and a coloring using $\chi(G)$ colors.



Problem 2. Prove that every planar graph G has a vertex v with $\deg_G(v) \leq 5$.

Problem 3. Delete one edge from the complete graph K_5. Find a planar drawing of the resulting graph.

Problem 4. Draw the labeled tree with Prüfer code 344567

Part B

In these problems, you will *select and justify appropriate tools to analyze a counting problem*. Two of these problems will be marked.

Problem 5. Figure 1 shows the Petersen Graph. Use Wagner's theorem (Theorem 5.8 in the notes for Lecture 12) to show that the Petersen Graph is nonplanar.

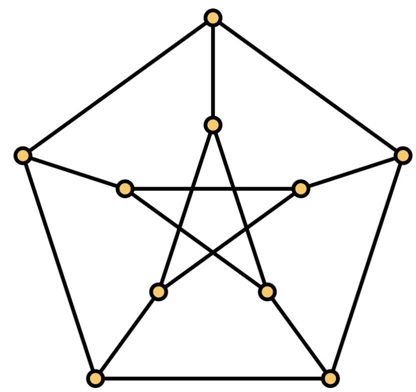


Figure 1: The Petersen Graph

Problem 6. Recall that the **degree sequence** of a graph is the list of the degrees of each vertex in decreasing order. Consider the following lists of six positive integers:

$$A = [3, 2, 2, 1, 1, 1]$$
$$B = [5, 5, 5, 4, 3, 2]$$
$$C = [5, 5, 4, 4, 3, 3]$$
$$D = [4, 4, 2, 2, 2, 2]$$
$$E = [3, 3, 3, 3, 3, 3]$$

Determine which sequence(s):

1. cannot be the degree sequence of a graph;
2. must be the degree sequence of an eulerian graph;
3. must be the degree sequence of a hamiltonian graph;
4. could be the degree sequence of a tree; and
5. could be the degree sequence of a graph, but cannot be a planar graph.

(As always, explain your answers).

Problem 7. For the next Olympic Winter Games, the organizers wish to expand the number of teams competing in curling. They wish to have 14 teams enter, divided into two pools of seven teams each. Right now, they're thinking of requiring that in preliminary play each team will play seven games against distinct opponents. Five of the opponents will come from their own pool and two of the opponents will come from the other pool. They're having trouble setting up such a schedule, so they've come to you. By using an appropriate graph-theoretic model, either argue that they cannot use their current plan or devise a way for them to do so.

Part C

In this problem, you will *construct counting problems which show the usefulness or limitations of combinatorial tools.* This problem will be marked for completeness only.

Problem 8. Give an example of a real life problem that can be modeled using planar graphs.