MAT344 Problem Set 2
(due 4am May 21)

Note: For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.

Part A

These questions relate to the learning outcomes *select and justify appropriate tools to analyze a counting problem* and *analyze a counting problem by proving an exact or approximate enumeration.* Two of these questions will be marked.

Problem 1. You are shopping for potatoes in the grocery store. There are red, white, purple and russet potatoes in stock.

(a) In how many ways can you buy 12 potatoes?

(b) In how many ways can you buy 12 potatoes if you insist of buying at least 2 purple and at least 1 white potato?

Problem 2. What is the coefficient of x^5y^2 in $(x+y+1)^9$?

Problem 3. Hockey games are played over three periods, and ties are broken by one overtime period. Suppose the Leafs tied the Canadiens 5-5 after regulation time. In how many ways could the score change over the game if the Leafs were never losing?

Problem 4. After expanding $(a+b+c+d)^7$ and combining like terms, how many terms are there? Justify your answer without performing the expansion.

Part B

These questions relate to the learning outcome *prove combinatorial identities by counting a set of objects in two ways*, *select and justify appropriate tools to analyze a counting problem* and *analyze a counting problem by proving an exact or approximate enumeration.* Two of these questions will be marked.

Remember: to give a combinatorial proof, you have to exhibit the set of objects you are counting and explicitly describe the two ways you are counting them.

Problem 5. Let n be a positive integer. Give a combinatorial proof of the identity

$$\sum_{i=0}^{n} i\binom{n}{i} = n2^{n-1}.$$

Problem 6. Let m, n be positive integers. Give a combinatorial proof of the identity

$$\sum_{i=0}^{n} \binom{m+i}{i} = \binom{m+n+1}{n}.$$

Problem 7. You want to walk 4 blocks North and 6 blocks East in a city with a grid of East-West and North-South streets.

(a) In how many ways can you do this?

(b) The intersection 2 blocks North and 2 blocks East (at $(2,2)$) from your starting point is blocked, so you can not pass through it. In how many ways can you walk to your destination now?

(c) Find the most inconvenient place for the block on the (East-West) street 2 streets North from your starting point, i.e. find the intersection $(k, 2)$ which, if blocked, results in the least number of available paths to your destination.

(d) Consider the intersection you found in the previous part. By moving it along the (North-South) street (to some other intersection (k, j)), can you make it more inconvenient?

(e) Can we conclude that the block at the most inconvenient place in the entire grid? Why or why not?

(f) **Challenge question (will not be graded):** Repeat parts (a)-(c) in the case where you have to walk 4 blocks North and n blocks East. *Hint: try to use Calculus.*

Part C

This question relates to the learning outcomes *construct counting problems which show the usefulness or limitations of combinatorial tools.* It will be marked for completeness only.

Problem 8. Give an example of a counting problem that can be solved using binomial coefficients.