MAT344 Problem Set 1
(due 4am May 14)

Note: For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.

Part A

These questions relate to the learning outcome *analyze a counting problem by proving an exact or approximate enumeration.* Two of these questions will be marked.

Problem 1. The Greek alphabet consists of 24 letters. How many seven-character strings can be made using the Greek alphabet?

Problem 2. In how many ways can you sit down 8 people around a round table if a family of 4 insists on being seated together?

Problem 3. In how many ways can you arrange 6 people to stand in a line if two people insist on not being next to each other?

Problem 4. A poker hand consists of five cards from a standard 52 card deck with four suits and thirteen values in each suit; the order of the cards in a hand is irrelevant. How many hands consist of 2 cards with one value and 3 cards of another value (a full house)? How many consist of 5 cards from the same suit (a flush)?

Part B

These questions relate to the learning outcomes *select and justify appropriate tools to analyze a counting problem* and *analyze a counting problem by proving an exact or approximate enumeration.* Two of these questions will be marked.

Problem 5. Pizza Pizza offers pizzas with the following options:

1. 4 sizes
2. 6 types of dough
3. 12 sauces (you can only select one of them, one of the 12 options is "no sauce")
4. 4 cheeses (you can only select one of them, one of the 4 options is "no cheese")
5. 19 types of veggie toppings (you may select any number of them)

6. 20 types of meat toppings (you may select any number of them)

7. 6 types of cheese toppings (and unlike with the "cheeses" above, you may select any number of them)

8. 6 types of "other" toppings (you may select any number of them)

Note: Pizza Pizza allows you to order double or triple orders of a single topping, let us just ignore this. In this problem, you are only allowed to either select a topping or not. Also, "no sauce" and "no cheese" are already included in the 12 sauces/4 cheeses, respectively. You will find that there are plenty of options already.

A group of friends eat ten Pizza Pizza pizzas for breakfast, lunch, and dinner every day. They insist on never eating the same pizza twice, and they have been doing this since the beginning of the universe (which you may assume to have been 13.799 billion years ago) then approximately what percentage of options have they exhausted as of today?

Problem 6. Let X be the set of the 26 lowercase English letters and 10 decimal digits. How many X-strings of length 15 satisfy all of the following properties (at the same time)?

- The first and last symbols of the string are distinct digits (which may appear elsewhere in the string).
- Precisely four of the symbols in the string are the letter 't'.
- Precisely three characters in the string are elements of the set $V = \{a, e, i, o, u\}$ and these characters are all distinct.

Problem 7. Jorge Luis Borges's "Library of Babel" is made up of hexagonal rooms. Each room has 5 bookshelves on 4 walls, and each shelf contains 32 books. Every book is 410 pages, with 40 lines on each page, and each line contains 80 characters. The characters are spaces, commas, periods, and 22 letters. If every book is unique, and the library contains every possible book, which is larger: the number of rooms in the library, or the largest known prime?

Part C

This question relates to the learning outcomes *construct counting problems which show the usefulness or limitations of combinatorial tools.* It will be marked for completeness only.

Problem 8. Give an example of a counting problem that can be solved using strings, permutations, or combinations.