MAT344 Problem Set 9
(due Thursday, Nov/28, noon)

Notes:

- For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.
- When submitting to Crowdmark, please ensure that your uploads are *legible*, *correctly rotated*, and *properly matched with the correct problems*. Any improperly uploaded problem scans will not be graded.
- Any assignments submitted after the deadline will not be accepted.

Part A

Three randomly chosen questions from this part will be marked.

Problem 1. Find an exponential generating function and the coefficient of $\frac{x^n}{n!}$ for the number of permutations with repetition of length n of the set $\{a, b, c\}$, in which there are an odd number of as, any number of bs, and an even number of cs.

Problem 2. Find an exponential generating function the number of partitions of $[n]$ into subsets of even size.

Problem 3. What is the coefficient of $x^n/n!$ in the exponential generating function $\frac{1}{1-2x}$?

Problem 4. Find the exponential generating function for the number of strings of length n formed from 26 uppercase English letters and 10 decimal digits if

- each vowel must appear at least one time (the vowels are “A”, “E”, “I”, “O”, “U”)
- the letter T must appear at least three times
- the letter Z may appear at most three times
- each even digit must appear an even number of times
- each odd digit must appear an odd number of times

Part B

Two randomly chosen questions from this part will be marked.

Problem 5.

(a) Find the exponential generating function for the number of ways of painting some number of different hotel rooms red, yellow and blue, if at most 2 can be painted red, an even number must be painted yellow, and any number can be painted blue.

(b) Use the generating function you found to answer the question for n rooms.

Problem 6. Find the exponential generating function (in closed form, not as an infinite sum) for each infinite sequence $\{a_n : n \geq 0\}$ whose general term is given below:

1. $a_n = 5^n$
2. $a_n = (-1)^n 2^n$
3. $a_n = 3^{n+2}$
4. $a_n = n!$
5. $a_n = n$
6. $a_n = \frac{1}{n+1}$

Problem 7.

(a) Find the closed form of the exponential generating function for the number of ways of creating a path graph on n vertices. (A path graph is a connected graph where two vertices have degree 1, and every other vertex has degree 2).

(b) Find the closed form of the exponential generating function for the number of labelled graphs on n vertices that can be partitioned into a number of path graphs.

Part C

This question will be marked for completion only.

Problem 8. Write an example of a counting problem that can be easily solved using exponential generating functions, but not ordinary generating functions.