MAT137 - Calculus with proofs

- Test 4 opens on March 12
- Assignment 9 due on March 25

- Today: Definition of series

- Wednesday: Properties of series
  Watch videos 13.5, 13.6, 13.7

Rapid questions: improper integrals

Convergent or divergent?

1. $\int_1^\infty \frac{1}{x^2}\,dx$
2. $\int_1^\infty \frac{1}{x}\,dx$
3. $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
4. $\int_1^\infty \frac{x+1}{x^3+2}\,dx$
5. $\int_1^\infty \frac{\sqrt{x^2+5}}{x^2+6}\,dx$
6. $\int_1^\infty \frac{x^2+3}{\sqrt{x^5+2}}\,dx$

A telescopic series

I want to calculate the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.

1. Find a formula for the k-th partial sum $S_k = \displaystyle\sum_{n=1}^{k} \frac{1}{n^2+2n}$.

   Hint: $\displaystyle \frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2}$

2. Using the definition of series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$$

What is wrong with this calculation? Fix it

Claim: $$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2$$

"Proof"

$$\begin{aligned}
\sum_{n=2}^{\infty} \ln \frac{n}{n+1} &= \sum_{n=2}^{\infty} [\ln n - \ln(n+1)] \\
&= \sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1) \\
&= (\ln 2 + \ln 3 + \ln 4 + \dots) - (\ln 3 + \ln 4 + \dots) \\
&= \ln 2
\end{aligned}$$

True or False – The tail of a series

1. IF the series $\sum_{n=0}^{\infty} a_n$ converges,

   THEN the series $\sum_{n=7}^{\infty} a_n$ converges

2. IF the series $\sum_{n=7}^{\infty} a_n$ converges,

   THEN the series $\sum_{n=0}^{\infty} a_n$ converges

3. IF the series $\sum_{n=0}^{\infty} a_n$ converges,

   THEN the series $\sum_{n=7}^{\infty} a_n$ converges to a smaller number.