

Math 157 Analysis I — Term Exam 3

University of Toronto, February 10, 2003

Name: _____ Student ID: _____

Solve the following 5 problems. Each is worth 20 points although they may have unequal difficulty. Write your answers in the space below the problems and on the front sides of the extra pages; use the back of the pages for scratch paper. Only work appearing on the front side of pages will be graded. Write your name and student number on each page. If you need more paper please ask the tutors. You have an hour and 50 minutes.

Allowed Material: Any calculating device that is not capable of displaying text.

Good Luck!

For Grading Use Only

1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

web version: <http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/TermExam3/Exam.html>

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Problem 1. Suppose that f is nondecreasing on $[a, b]$. Notice that f is automatically bounded on $[a, b]$, because $f(a) \leq f(x) \leq f(b)$ for any x in $[a, b]$.

1. If $P = \{t_0, \dots, t_n\}$ is a partition of $[a, b]$, write formulas for $L(f, P)$ and $U(f, P)$ in as simple terms as possible.
2. Suppose that $t_i - t_{i-1} = \delta$ for each i . Show that $U(f, P) - L(f, P) = \delta[f(b) - f(a)]$.
3. Prove that f is integrable.

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Problem 2. In each of the following, f is a continuous function on $[0, 1]$.

1. Show that

$$\int_0^\pi f(\sin x) \cos x \, dx = 0.$$

2. Characterize the functions f that have the property that

$$\int_0^x f(t) \, dt = \int_x^1 f(t) \, dt \quad \text{for all } x \in [0, 1].$$

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Problem 3.

1. Prove that if two functions f_1 and f_2 both satisfy the differential equation $f'' + f = 0$ and if they have the same value and the same first derivative at 0, then they are equal.
2. Use the above to show that $\sin(\frac{\pi}{2} + x) = \cos x$ for all x . (Do not use the formula for the sin of a sum!)

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Problem 4.

1. Compute

$$\lim_{x \rightarrow 0} \frac{e^x - (1 + x)}{x^2}.$$

2. Use your result to estimate the difference between $e^{0.1}$ and 1.1. Warning: a 10 digit answer obtained with your calculator may contribute negatively to your grade. You shouldn't use any calculating device and your derivation of the answer should be simple enough that it be clear that you didn't need any machine help.

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Problem 5. Evaluate the following integrals in terms of elementary functions:

1. $\int x^2 \cos x \, dx$

2. $\int \frac{dx}{x^2 - 3x + 2}$ (cancelled)

3. $\int_0^1 \frac{dx}{1 + e^x}$

4. $\int_0^\infty x e^{-x^2} \, dx$

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