Homework Assignment 22

Assigned Tuesday March 11; due Friday March 21, 2PM at SS 1071

web version: http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/HW22/HW22.html

Required reading. All of Spivak Chapter 23. Also read (but don't do!) all exercises for that chapter — just to get an impression for how intricate the various convergence tests and criteria can get.

To be handed in. From Spivak Chapter 23: 1 (parts divisible by 4), 12, 23 as well as the following question:

- Prove that the following sums diverge: (Hint: Use problem 20.)

$$\sum_{n=1}^{\infty}\frac{1}{n};\qquad \sum_{n=2}^{\infty}\frac{1}{n(\log n)};\qquad \sum_{n=3}^{\infty}\frac{1}{n(\log n)(\log\log n)};$$

$$\sum_{n=16}^{\infty}\frac{1}{n(\log n)(\log\log n)(\log\log\log n)};\quad \dots$$

- Prove that the following sums converge: (Hint: Use problem 20.)

$$\sum_{n=1}^{\infty}\frac{1}{n^{1.01}};\qquad \sum_{n=2}^{\infty}\frac{1}{n(\log n)^{1.01}};\qquad \sum_{n=3}^{\infty}\frac{1}{n(\log n)(\log\log n)^{1.01}};$$

$$\sum_{n=16}^{\infty}\frac{1}{n(\log n)(\log\log n)(\log\log\log n)^{1.01}};\quad \dots$$

Recommended for extra practice. From Spivak Chapter 23: 1 (the rest), 5, 20, 21 as well as the following question:

- In this question we always assume that $a_n > 0$ and $b_n > 0$. Let's say that a sequence a_n is "much bigger" than a sequence b_n if $\lim_{n\to\infty} a_n/b_n = \infty$. Likewise let's say that a sequence a_n is "much smaller" than a sequence b_n if $\lim_{n\to\infty} a_n/b_n = 0$. Prove that for every convergent series $\sum b_n$ there is a much bigger sequence a_n for which $\sum a_n$ is also convergent, and that for every divergent series $\sum b_n$ there is a much smaller sequence a_n for which $\sum a_n$ is also divergent. (Thus you can forever search in vain for that fine line between good and evil; it just isn't there).