

Dror Bar-Natan: Classes: 2002-03: Math 157 - Analysis I:

Homework Assignment 21

Assigned Tuesday March 4; due Friday March 14, 2PM at SS 1071

web version: <http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/HW21/HW21.html>

Required reading. All of Spivak Chapter 22.

To be handed in. From Spivak Chapter 22: 1 (even parts), 2 (even parts), 5, 13.

Recommended for extra practice. From Spivak Chapter 22: 1 (odd parts), 2 (odd parts), 9, 27, 28, 29.

Just for fun. For some constant number c , consider the function $f_c(x) = 4cx(1 - x)$. Let A be the set of all pairs (c, y) so that $0 \leq c \leq 1$ and y is a limit of a subsequence of the sequence $f_c(\frac{1}{2}), f_c(f_c(\frac{1}{2})), f_c(f_c(f_c(\frac{1}{2}))), \dots$. Write a computer program to draw the set A in the plane whose axes are c and y , and if your program and picture are nice, they'll find their place on this class' web site.

It's a hard one, but it's well worth it. The set A is way more complex than you would expect, with parts that scream "structure" and parts that scream "mess". If you've ever heard the word "chaos" in a mathematical context before, this is it. And if you've ever seen pictures of the beautiful "Mandelbrot Set", our A is a close relation.