Homework Assignment 8

Assigned Tuesday October 29; due Friday November 8, 2PM at SS 1071

web version: http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/HW08/HW08.html

Required reading

All of Spivak Chapter 9.

To be handed in

From Spivak Chapter 9: 1, 9, 15, 23.

Recommended for extra practice

From Spivak Chapter 9: 8, 11, 21, 28.

Also, let $p(x)$ be the polynomial $x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$. Now that we know that for $|x| > 2n\max(|a_{n-1}|, \ldots, |a_1|, |a_0|, 1)$ we have that

$$\frac{1}{2}|x^n| > |a_{n-1}x^{n-1} + \cdots + a_1 x + a_0|,$$

complete the proof of the following

Theorem.

- *If n is odd then the equation $p(x) = c$ has a root for any value of c.*
- *If n is even then there is some constant c_0 so that the equation $p(x) = c$ has no roots for $c < c_0$, has at least one root for $c = c_0$ and at least two roots for $c > c_0$.*

Just for fun

Write a computer program that will allow you to draw the graph of the function

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \sin 3^n x,$$

and will allow you to zoom on that graph through various small "windows". Use your program to convince yourself that f is everywhere continuous but nowhere differentiable. The best plots will be posted on this web site! (Send pictures along with window coordinates by email to drorbn@math.toronto.edu).