Homework Assignment 1

assigned Sep. 10; due Sep. 24, 2PM at SS 1071

Required reading

All of Spivak Chapter 1.

To be handed in

From Spivak Chapter 1:

Hand in	**Don't hand in**
11 even parts	7
12 even parts	15
14	18, 20

And also (to be handed in)

1. Show that if $a > 0$, then $ax^2 + bx + c \geq 0$ for all values of x if and only if $b^2 - 4ac \leq 0$.
2. Prove the Cauchy-Schwartz inequality

$$\left(a_1b_1 + a_2b_2 + \cdots a_nb_n\right)^2 \;\leq\; \left(a_1^2 + \cdots + a_n^2\right)\left(b_1^2 + \cdots + b_n^2\right)$$

in two different ways:

(a) Use $2xy \leq x^2 + y^2$ (why is this true?), with

$$x = \frac{|a_i|}{\sqrt{a_1^2 + \cdots + a_n^2}} \qquad y = \frac{|b_i|}{\sqrt{b_1^2 + \cdots + b_n^2}}$$

(b) Consider the expression

$$(a_1x + b_1)^2 + (a_2x + b_2)^2 + \cdots + (a_nx + b_n)^2,$$

collect terms, and apply the result of Problem 1.

Recommended for extra practice

Spivak Chapter 1: 21, 22, 23