1. List all tournaments with five vertices, up to isomorphism. Prove the list is complete.

2. Let $G$ be a simple connected graph with $n$ vertices and let $v, w$ be two non-connected vertices in $G$ so that $d(v) + d(w) \geq n$. Let $H$ be the graph constructed from $G$ by adding a new edge $uv$. Prove that $G$ has a hamilton cycle iff $H$ has one.

3. Let $G$ be a loopless graph each of whose vertices has valence $\geq 3$. Prove that $G$ has a cycle of even length.

4. $m$ identical pizzas are to be shared equally amongst $n$ students.
   (a) Show how this goal can be achieved by dividing the pizzas into a total of $m + n - d$ pieces, where $d$ is the greatest common divisor of $m$ and $n$.
   (b) By considering a suitable bipartite graph, show that no division into a smaller number of pieces will achieve the same objective.

5. Show that a connected graph of diameter $d$ has a spanning tree of diameter at most $2d$. 