Math 246Y: Homework number 1
Due Tuesday, October 14, 2003 at 6:10 PM sharp!

(1) We know that multiplication of integers is commutative; i.e. \( ab = ba \)
for all pairs of integers \( a \) and \( b \). Prove that, for every natural number \( n \), the product of \( n \) integers is independent of the order of the factors.

(2) Is \( 3^{5729} - 5 \) divisible by 10? Prove that your answer is correct.

(3) Prove that \( 4 \times 7^{2003} + 3 \) is divisible by at least one prime that leaves remainder 3 upon division by 4 (i.e. this prime is of the form \( 4n + 3 \) for some \( n \in \mathbb{Z} \)).

(4) Suppose that \( p \) is a prime and \( a \) and \( k \) are any two positive integers. Prove that \( a^p \) is congruent to \( a \) modulo \( p \).

(5) Suppose that \( a \) is an integer. Show that if \( a^3 \) is of the form \( 4n + 3 \) (for some \( n \in \mathbb{Z} \)), then \( a \) is of the form \( 4m + 3 \) (for some \( m \in \mathbb{Z} \)). Hint: Congruences can be useful here.