Math 327, Term Test
Fall Quarter 2011
Thu, Oct 27

Name: ______________________

Instructions:
Show all your work on these sheets. Justify your answers! This test has 5 problems and 7 pages. Make sure you have all of them. No calculators, books, notes, etc. are allowed.

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Question 1. (10 points)

(a) Define the concept of a basis for a topology on a set $X$.

(b) Is the collection of all closed intervals, $C = \{[a,b] : a < b \text{ in } \mathbb{R}\}$, a basis for a topology on the set of real numbers?

(c) Suppose that $X, Y$ are topological spaces. Show that $\{U \times V : U \text{ open in } X, V \text{ open in } Y\}$ is a basis for a topology on $X \times Y$. You are not allowed to assume anything about the product topology!
Question 2. (10 points)

(a) Show that $\mathbb{R}_\ell$ is disconnected.

(b) Show that if $X$ is path-connected, then $X$ is connected.

(c) Is $\mathbb{R}_\ell$ path-connected?
Question 3. (10 points) Suppose that $A \subset X$ and suppose that $f : A \rightarrow Y$ is a continuous function with $Y$ Hausdorff. Show that there is at most one continuous function $g : \overline{A} \rightarrow Y$ that extends $f$ (that is, $g|_A = f$).
Question 4. (10 points) Recall that, as a set, \( \mathbb{R}^\omega = \prod_{n=1}^{\infty} \mathbb{R} \).

(a) Is the function \( f : \mathbb{R} \to \mathbb{R}^\omega \) with \( f(t) = (t, 2t, 3t, \ldots) \) continuous, if \( \mathbb{R}^\omega \) carries the product topology?

(b) Does the sequence in \( \mathbb{R}^\omega \),

\[
\begin{align*}
x_1 &= (1, 0, 0, 0, \ldots) \\
x_2 &= \left(\frac{1}{2}, \frac{1}{2}, 0, 0, \ldots\right) \\
x_3 &= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots\right) \\
& \vdots
\end{align*}
\]

converge, if \( \mathbb{R}^\omega \) carries the box topology? Justify your answer.
Question 5. (10 points) Suppose that $(X, d)$ is a metric space. Fix any element $x \in X$ and $r > 0$, a positive real number.

(a) Show that $C_r(x) = \{y \in X : d(x, y) \leq r\}$ is a closed subset of $X$.

(b) Give an example to show that $B_r(x)$ need not be equal to $C_r(x)$. (Recall that $B_r(x) = \{y \in X : d(x, y) < r\}$. Hint: one can find such an example that is a subset of $\mathbb{R}$ with the induced metric.)
Scratch Paper