From this picture (drawn with help from Jacob Tsimerman) we can read the following:

$$
\alpha_1^\dagger = a_1^* + r_1^* + r_2^*, \quad -\beta_1^\dagger = b_1^* + r_3^* + r_2^*.
$$

$$
\partial_2 A_{1+} = r_0, \quad \partial_0 A_{1+} = a_1, \quad \alpha_1^\dagger \cup \beta_1^\dagger(A_{1+}) = \alpha_1(\partial_2 A_{1+}) \beta_1(\partial_0 A_{1+}) = 0 \cdot 0 = 0.
$$

$$
\partial_2 B_{1+} = r_1, \quad \partial_0 B_{1+} = b_1, \quad \alpha_1^\dagger \cup \beta_1^\dagger(B_{1+}) = \alpha_1(\partial_2 B_{1+}) \beta_1(\partial_0 B_{1+}) = 1 \cdot (-1) = -1.
$$

$$
\partial_2 A_{1-} = r_3, \quad \partial_0 A_{1-} = a_1, \quad \alpha_1^\dagger \cup \beta_1^\dagger(A_{1-}) = \alpha_1(\partial_2 A_{1-}) \beta_1(\partial_0 A_{1-}) = 0 \cdot 0 = 0.
$$

$$
\partial_2 B_{1-} = r_4, \quad \partial_0 B_{1-} = b_1, \quad \alpha_1^\dagger \cup \beta_1^\dagger(B_{1-}) = \alpha_1(\partial_2 B_{1-}) \beta_1(\partial_0 B_{1-}) = 0 \cdot (-1) = 0.
$$

So $\alpha_1^\dagger \cup \beta_1^\dagger = -B_{1+}^*$ is a generator of $H^2$.

**Exercise.** Verify that $\beta_1^\dagger \cup \alpha_1^\dagger = -A_{1-}^*$ is also a generator of $H^2$, but note that in $H^2$ we have $B_{1+}^* = -A_{1-}^*$ so the cup product is not commutative!

**The Hopf Fibration** as drawn by Penrose and Rindler: