Homework Assignment 5

Assigned Thursday October 16; due Thursday October 23 in class.

Required reading. Sections 1 and 2 of my paper On the Vassiliev Knot Invariants.

To be handed in.

1. Let $\Delta$ be the “doubling” (also called “cabling”) operation on knots, which takes a framed knot and replaces it by a 2-component link by “replacing every line by a double line” in an obvious manner.

   (a) Show that if $V$ is a type $m$ invariant of 2-component links then $V \circ \Delta$ is a type $m$ invariant of knots.

   (b) Find a map $\Delta : A(\otimes) \to A(\otimes\otimes)$ (sorry for the “operator overloading”) for which $W_{V \circ \Delta} = W_V \circ \Delta$ for all such $m$ and $V$. (Verify that you proposed map respects the 4T relation!)

2. If $D$ is a chord diagram, let $X(D)$ be the number of “chord crossings” in $D$ (so for example, $X(\otimes) = 1$).

   (a) Does $X : D \to \mathbb{Z}$ satisfy the 4T relation?

   (b) Let $m$ by a natural number. Can you find a type $m$ knot invariant $V$ for which $W_V = X$?

Idea for a good deed. Tell us about the Milnor-Moore theorem: A connected commutative and co-commutative graded Hopf algebra over a field of characteristic 0 which is of finite type, is the symmetric algebra over the vector space of its primitives.